Imperfect Repair Systems: Test and Model Selection

Enrico A. Colosimo
(UFMG, Brazil)

Joint work: Gustavo Gilardoni (UnB), Maria Luiza Toledo (ENCE-IBGE) and Marta Freitas (UFMG), Brazil

Institut Polytechnique de Grenoble - Laboratoire Jean Kuntzmann
Université Grenoble
July 1*st*, 2016

Research Team




- Prof. Gustavo Gilardoni (UnB)
- Profa. Maria Luiza Toledo (ENCE-IBGE)
- Postdoctoral researcher - Rodrigo Reis (UFMG)
- Profa. Marta Freitas (UFMG)

Brazil

RORAIMA
Boa Vista
Amajari
Caracaraí
Normandia
Caroebe
AMAPÁ
Oiapoque
Calçoene
Laranjal do Jari
Serra do Navio
Macapá
Porto Santana
AMAZONAS
Içana
Tapurucuara
Japurá
Coari
Manaus
Itacoatiara
Parintins
Tabatinga
Atalaia do Norte
Tapauá
Maués
Humaitá
Prainha
Lábrea
PARÁ
Oriximiná
Almeirim
Óbidos
Santarém
Ilha de Marajó
Belém
Abaetetuba
Cametá
Afuá
Tucuruí
Altamira
Itaituba
Paragominas
Marabá
São Félix
Jacareacanga
Conceição do Araguaia
Cachimbo
Salinópolis
Bragança
ACRE
Mâncio Lima
Tarauacá
Feijó
Rio Branco
Sena Madureira
RONDÔNIA
Porto Velho
Ji-Paraná
Nova Mamoré
Guajará-Mirim
Costa Marques
Vilhena
MARANHÃO
São Luís
Alcântara
Rosário
Monção
Chapadinha
Araguanã
Codó
Imperatriz
Carolina
Balsas
Buriti Bravo
PIAUÍ
Teresina
Picos
Oeiras
Campo Maior
São Raimundo Nonato
Guaribas
Corrente
CEARÁ
Acaraú
Itapipoca
Caucaia
Fortaleza
Aracati
Sobral
Quixadá
Mombaça
Juazeiro do Norte
Crato
RIO GRANDE DO NORTE
Mossoró
Natal
Caicó
PARAÍBA
Souza
Patos
Campina Grande
João Pessoa
PERNAMBUCO
Serra Talhada
Caruaru
Olinda
Recife
Jaboatão
Garanhuns
Petrolina
ALAGOAS
Maceió
Arapiraca
SERGIPE
Propriá
Aracaju
TOCANTINS
Nazaré
Araguaína
Goiatins
Palmas
Porto Nacional
Sandolândia
Gurupi
Arraias
BAHIA
Juazeiro
Canudos
Barra
Senhor do Bonfim
Lagarto
Barreiras
Macaúbas
Feira de Santana
Alagoinhas
Bom Jesus da Lapa
Jequié
Itabuna
Salvador
Ilhéus
Vitória da Conquista
Brumado
Itarantim
Canavieiras
Porto Seguro
MATO GROSSO
Colniza
Apiacás
Nova Bandeirantes
Matupá
Juara
Juína
Feliz Natal
Querência
Sapezal
Comodoro
Ribeirão Cascalheira
Paranatinga
Chapada dos Guimarães
Barra do Garças
Cuiabá
Cáceres
Alto Garças
Poconé
Rondonópolis
GOIÁS
Minaçu
Cavalcante
Crixás
Niquelândia
Goiás Velho
Brasília
Anápolis
Caiapônia
Goiânia
Catalão
Caldas Novas
Jataí
MINAS GERAIS
São Francisco
Januária
Montes Claros
Jequitinhonha
João Pinheiro
Teófilo Otoni
Curvelo
Guaratinga
Itamaraju
Gov. Valadares
Uberlândia
Araxá
Belo Horizonte
Uberaba
Divinópolis
Ouro Preto
São João del Rei
Barbacena
Guaxupé
Juiz de Fora
Poços de Caldas
ESPÍRITO SANTO
Linhares
Colatina
Cariacica
Vitória
Vila Velha
Cachoeiro do Itapemirim
MATO GROSSO DO SUL
Coxim
Corumbá
Água Clara
Bonito
Dourados
Campo Grande
Ponta Porã
Amambai
SÃO PAULO
S. J. do Rio Preto
Araçatuba
Franca
Ribeirão Preto
Pres. Prudente
Araraquara
Marília
S. J. dos Campos
Campinas
São Paulo
Taubaté
Santos
Ubatuba
RIO DE JANEIRO
Nova Friburgo
Campos
Rio de Janeiro
Resende
Volta Redonda
PARANÁ
Paranavaí
Londrina
Assaí
Maringá
Goio-Erê
Campo Mourão
Foz do Iguaçu
Ponta Grossa
Curitiba
Guarapuava
União da Vitória
Paranaguá
Iguape
Cananéia
SANTA CATARINA
Chapecó
Joinville
Blumenau
Treze Tílias
Camboriú
Lages
Florianópolis
Criciúma
Laguna
Araranguá
RIO GRANDE DO SUL
Erechim
Passo Fundo
Gramado
Sta. Maria
Caxias do Sul
Alegrete
Porto Alegre
Torres
Bagé
Pelotas
Rio Grande
Sta. Vitória do Palmar
Chuí
220 km 0 220 440 km

Outline

- Motivating Situation: Dump Trucks Data
- Counting Process Formulation
- Non-parametric Test for Imperfect Repair
- ARA and ARI Classes of IR Models
- Models Selection/Reliability Prediction
- Dump Trucks Data Revisited
- Final Remarks

Motivating situation

Mining DUMP truck

Motivating situation

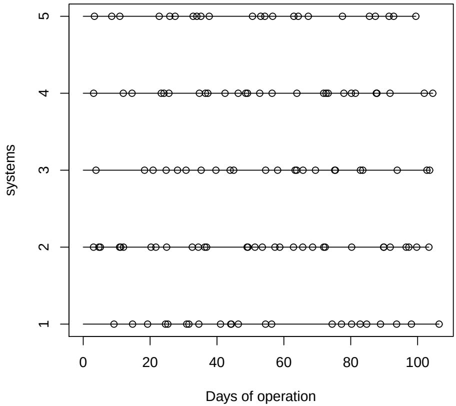
- ✓ Designed to operate in road conditions.
- ✓ DUMP trucks are used to transport mining production.
- ✓ In the mine under study, they are used in much more severe conditions.
- ✓ Engineering interest: engine failures.

Motivating Situation (cont.)

DATA SET

- ✓ Data colection in the period: july to october, 2012, for 5 trucks;
- ✓ the accumulated number of working days were registered;
- ✓ a total of 129 failures were observed, each one followed by a repair;

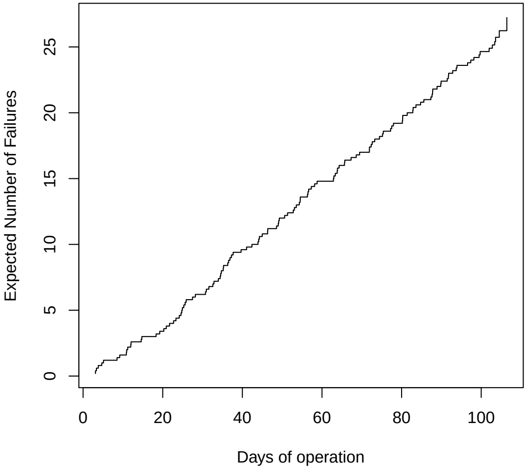
Motivating situation (cont.)



Failure times in days of operation ("o" are failures).

Motivating situation (cont.)

Empirical Mean Cumulative Function (MCF)



Motivating situation (cont)

OBJECTIVES

- ✓ Estimate the effect of the repairs and the aging speed of the engine's trucks.
- ✓ Estimate reliability predictors to provide information in order to base the decision-making process related to maintenance policies.

Motivating situation (cont)

Specific Objectives

✓ Is the repair imperfect or mininum?

✓ Reliability Prediction:

- Model selection.
- Estimate repair and age effects and reliability Predictors.

General modeling of a counting process

- $N(t)$: number of observed failures up to time t;
- $\{N(t)\}_{t\geq 0}$: counting processes that is characterized by the failure intensity function (Andersen et.al, 1993):

$$\lambda(t) = \lim_{h\to 0} \frac{P(N(t+h) - N(t) = 1|\Im_t)}{h}, \quad \forall t \geq 0$$

where $\Im_t$ is the history of the process up to time t $(T_1 = t_1, \ldots, T_{N(t)} = t_{N(t)})$.

General modeling of a counting process (cont.)

- Cumulative Intensity: $\Lambda(t) = \int_0^t \lambda(u)du$.
- Mean Cumulative Intensity: $\Phi(t) = E(N(t)) = E[\Lambda(t)]$, (Aalen, 1978).
- ROCOF (rate of occurrence of failures) function is

$$\phi(t) = \Phi'(t) = E[\lambda(t)]$$

Repair Types

1. Perfect Repair (PR)
    - system leaves as it were new (As Good As New).
    - Renew Process.
2. Minimal Repair (MR)
    - system stays in the same condition as before the failure (As Bad As Old).
    - NHPP: $\phi(t) = \lambda(t)$.
3. Imperfect Repair
    - system leaves in a condition between ABAO and AGAN.
    - Probably the one appropriate for the Mining Dump Truck, according to the maintenance engineers.

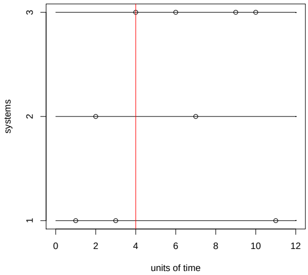
Non-Parametric Test for Minimum Repair

$$H_0 : \text{MR} \quad \text{vs} \quad H_1 : \text{Non Harmful Repair/First Order}$$

It means:

- Non Harmful Repair: System reliability improves after repair.
- First Order: $\lambda(t)$ just depend on the last failure, $t_{N(t)}$.

Non-Parametric Test Idea



- Under H_0: MR, systems are a homogeneous NHPPs sample, with an increasing $\lambda(t)$ in t.
- Under H_0: MR, each system would have the same probability to be the next failure's system.
- Under H_1: no harmfull/first order repair, failure system will take longer time for the next failure as compared with the others.

Notation and Assumptions

- k identical repairable systems, where the failures history occurs independently;
- at each failure, a repair action of negligible length is performed;
- n_i failures are observed in the i^{th} system, $i = 1, 2, \ldots, k$;
- $N = \sum_{i=1}^{k} n_i$ is the total number of observed failures in the systems.
- Let r_l; $l = 1, \ldots, N$, be the rank of the observed failure times in the overall sample; $r_1 < r_2 < \ldots < r_N$.
- $Z_{i,l} = 1\ (G_l = i)$, for the systems membership.

Non-Parametric Test for Minimum Repair

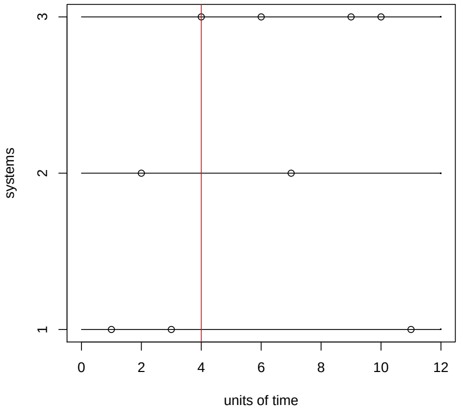
- Let´s define,

$$X_l = \begin{cases} 1, \ Z_{i,l} = Z_{i,l+1}, \ \text{for all} \ \ i = 1, \dots, k \\ 0, \ \ \text{otherwise} \end{cases} \tag{1}$$

  for $l = 1, \dots, N - 1$.

- Let´s define the test statistic: $T = \sum_{l=1}^{N-1} X_l$.

Non-Parametric Test Idea



$$X_1 = X_2 = X_3 = X_5 = X_6 = X_8 = 0 \text{ and } X_4 = X_7 = 1 \text{ and } T = 2.$$

Non-Parametric Test

- Therefore, under H_0, T has a bin$(N - 1, \pi = 1/k)$.
- Test can be reformulated in the following terms:

$$H_0 : \pi = \frac{1}{k} \text{ vs } H_1 : \pi < \frac{1}{k}.$$

- For an observed T=t, p-value = $P(T \leq t|\pi = 1/k)$.

Non-Parametric Test for Minimum Repair

Small size Monte Carlo Simulation

- Descriptive size evaluation of the test
- Scenarios
  - Number of systems: K=5,10;
  - Truncation times: T=5,10,15;
  - Power Law Process

$$\lambda(t) = \frac{\beta}{\eta} \left\{ \frac{t}{\eta} \right\}^{\beta-1}$$

  - $\eta = 1$ and $\beta = 1.5, 2,$
- 10000 replicates.

Non-Parametric Test for Imperfect Repair

Tabela: Monte Carlo Simulation Results

Scenario			Covarage		$\widehat{\Phi}$/System
			$\alpha = 0.05$	$\alpha = 0.10$	
$K = 5$	$T = 5$	$\beta = 1.5$	0.031	0.068	11
		$\beta = 2$	0.044	0.086	25
	$T = 10$	$\beta = 1.5$	0.039	0.085	31
		$\beta = 2$	0.046	0.090	100
	$T = 15$	$\beta = 1.5$	0.045	0.085	58
		$\beta = 2$	0.048	0.091	225
$K = 10$	$T = 5$	$\beta = 1.5$	0.035	0.074	11
		$\beta = 2$	0.039	0.083	25
	$T = 10$	$\beta = 1.5$	0.040	0.082	32
		$\beta = 2$	0.045	0.087	100
	$T = 15$	$\beta = 1.5$	0.044	0.090	58
		$\beta = 2$	0.046	0.095	225

- Dump trucks data: p-value = 0.08544. It is an indication against the hypothesis of minimum repair.

Imperfect Repair (IR) Models

- Kijima *et al.* (1988) and Kijima (1989)
  - Virtual age models: rejuvenate the system after repair;
  - the degree of efficiency of the repair is represented by θ $(0 \leq \theta \leq 1)$.
- Doyen and Gaudoin (2004): ARA and ARI classes of models.
  - start at $t = 0$ with a initial intensity $\lambda_R(t)$.
  - has a rule to define how intensity changes immediately after a failure/repair.

ARA class of models (Doyen and Gaudoin, 2004)

- ARA_m model: repair reduces the increment in system age since the last m failures (memory parameter). Its failure intensity function is expressed by

$$\lambda_{ARA_m}(t) = \lambda_R\left(t - (1-\theta) \sum_{j=0}^{Min(m-1,N(t)-1)} \theta^j T_{N(t)-j}\right).$$

- Virtual age model proposed by Kijima *et al.* (1988) corresponds to the ARA_1 $(m = 1)$.

ARI class of models (Doyen and Gaudoin, 2004)

- Repair action reduces the failure intensity function of the system;
- repair reduces the increment in failure intensity since the last *m* failures;
- Its failure intensity function is expressed by:

$$\lambda_{ARI_m}(t) = \lambda_R(t) - (1-\theta) \sum_{j=0}^{Min(m-1,N(t)-1)} \theta^j \lambda_R(T_{N(t)-j}).$$

Likelihood Function for a Counting Processes

- a counting process $N(t)$ observed from 0 until a time T, $0 = t_0 < t_1 < \ldots < t_{N(T)} \leq t^*$
- Lindqvist (2006), Andersen et al. (1993).
- ARA and ARI models.

Likelihood function for the ARA class of models

$$L_{ARA_m}(\mu) =$$

$$= \prod_{i=1}^{k} \prod_{j=1}^{n_i} \{\lambda_R(t_{i,j} - (1-\theta) \sum_{p=0}^{min(m-1,j-2)} \theta^p t_{i,j-1-p}) \times$$

$$\times e^{-\Lambda_R(t_{i,j}-(1-\theta)\sum_{p=0}^{min(m-1,j-2)} \theta^p t_{i,j-1-p})+\Lambda_R(t_{i,j-1}-(1-\theta)\sum_{p=0}^{min(m-1,j-2)} \theta^p t_{i,j-1-p})}\} \times$$

$$\times e^{-\Lambda_R(t_i^*-(1-\theta)\sum_{p=0}^{min(m-1,n_i-1)} \theta^p t_{i,n_i-p})+\Lambda_R(t_{i,n_i}-(1-\theta)\sum_{p=0}^{min(m-1,n_i-1)} \theta^p t_{i,n_i-p})}$$

Likelihood function for the ARI class of models

$$L_{ARI_m}(\mu) =$$

$$= \prod_{i=1}^{k} \prod_{j=1}^{n_i} \{[\lambda_R(t_{i,j}) - (1-\theta) \sum_{p=0}^{min(m-1,j-2)} \theta^p \lambda_R(t_{i,j-1-p})] \times$$

$$\times e^{-\Lambda_R(t_{i,j}) + \Lambda_R(t_{i,j-1}) + (1-\theta)[t_{i,j} - t_{i,j-1}] \sum_{p=0}^{min(m-1,j-2)} \theta^p \lambda_R(t_{i,j-1-p})}\} \times$$

$$\times e^{-\Lambda_R(t_i^*) + \Lambda_R(t_{i,n_i}) + (1-\theta)[t_i^* - t_{i,n_i}] \sum_{p=0}^{min(m-1,n_i-1)} \theta^p \lambda_R(t_{i,n_i-p})},$$

Parameter Estimation in ARA and ARI models

For both cases, the likelihood function was rewritten assuming a PLP parametric form for the initial intensity :

$$\lambda_R(t) = \frac{\beta}{\eta} \left\{ \frac{t}{\eta} \right\}^{\beta-1}$$

therefore, $\mu = (\beta; \eta; \theta)$ is the vector of parameters to be estimated.

Model selection

- maximum value of the estimated likelihoods: $\hat{L} = L(\hat{\theta}; \hat{\beta}; \hat{\eta})$

- Burnham and Anderson (2004) : weight of evidence in favor of model r, given by:

$$w_r = \frac{exp(-\Delta_r/2)}{\sum_{r=1}^{R} exp(-\Delta_r/2)}$$

where

- $\Delta_r = \hat{L}_{max} - \hat{L}_r, \quad (r = 1, \ldots, R)$

- and $\hat{L}_{max}$ is the maximum of the R different $\hat{L}$ values, considering that R different models were fitted.

This transformation forces the best model to have $\Delta = 0$, while the rest of the models have positive values.

Model selection - Goodness-of-fit plot

- MCF $\Phi(.)$ is estimated to each model according to these steps:
  - ML estimates are obtained for a model from its observed failure data.
  - Observed failure data for the i-th system ($i = 1, \ldots, k$) and the MLEs are plugged in the model intensity function, providing $\hat{\lambda}_i(t)$.
  - $\hat{\Phi}_i(t)$ is computed as $\int_0^t \hat{\lambda}_i(u)du$, for $0 \leq t \leq t_i^*$.
  - Finally, $\hat{\Phi}(t)$ is obtained as $\frac{\sum_{i=1}^{K} \hat{\Phi}_i(t)}{K}$.
- Plot of $\hat{\Lambda}(t)$ against empirical MCF (Nelson-Aalen plot).

Dump trucks data set revisited

The following models were considered:

- ARA_m; $m = 1, \dots, 31$
- ARI_m; $m = 1, \dots, 31$
- Minimal Repair ($\theta = 1$)

In cases 1) and 2), $m = 31$ corresponds to $m = \infty$ (max number of failures) and the PLP parametric form for the initial intensity was adopted.

Tabela: Results of the model fitting - Dump trucks data

Estimated values	MODELS			
	MR	ARA_1	ARA_{13}	ARA_∞
$\hat{\beta}$	1.14	1.33	1.80	1.81
	[0.96;1.35]	[1.05;1.69]	[1.39;2.35]	[1.39;2.35]
$\hat{\eta}$	5.92	4.94	7.58	7.59
	[3.54;9.93]	[3.41;7.15]	[5.35;10.76]	[5.35;10.79]
$\hat{\theta}$	-	0.02	0.60	0.60
	-	[0.0001;0.53]	[0.43;0.84]	[0.42;0.84]
$\hat{L}$	-307.1811	-304.7039	-300.3218	-300.3165
		ARI_1	ARI_{13}	ARI_∞
$\hat{\beta}$		1.42	1.89	1.90
		[1.06;1.91]	[1.69;2.11]	[1.71;2.11]
$\hat{\eta}$		4.18	7.48	7.65
		[2.57; 6.79]	[5.74; 10.11]	[5.76;10.17]
$\hat{\theta}$		0.23	0.67	0.67
		[0.05;1.00]	[0.52;0.87]	[0.52;0.87]
$\hat{L}$		-306.2146	-300.0904	-300.1155

Dump trucks data set revisited (cont.)

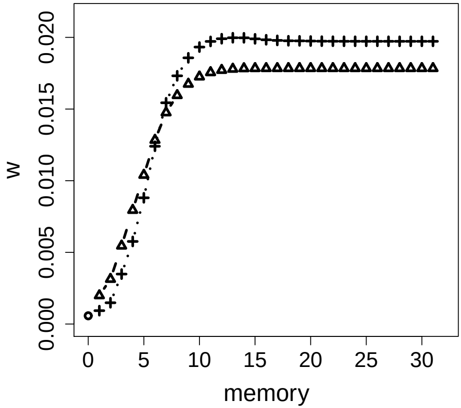
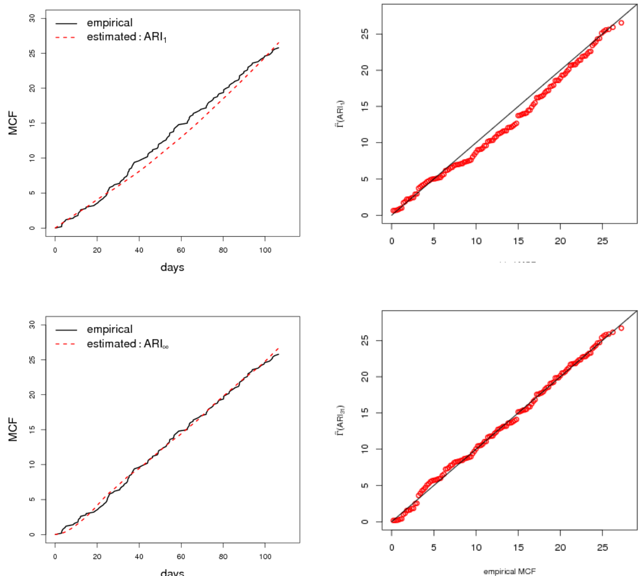


Figura: Criterion value for model selection (0 - MR, + ARI and Δ - ARA).

Dump trucks data revisited (cont.) - GOF Plots



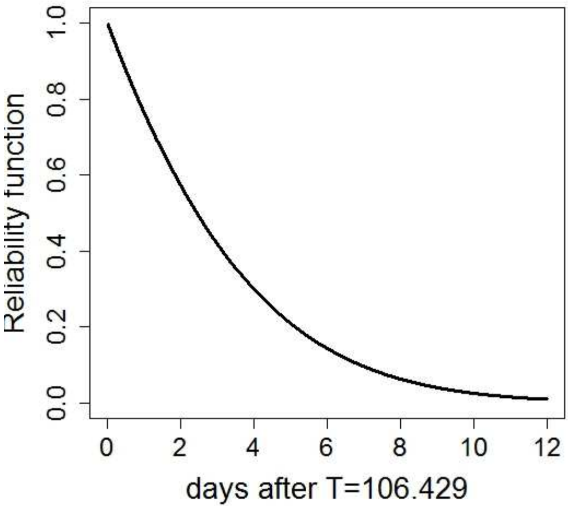
Dump trucks data revisited (cont.)

- for ARI_∞, $\hat{\beta} = 1.90$ ([1.71; 2.11]) , indicating that the equipment failure intensity function increases with time (intrinsic aging)
- $\hat{\theta} = 0.67$ ([0.52,0.87]). The repairs after failures tend to leave the equipament in a state between **AGAN and ABAO**

Reliability Prediction Functions - (ARI_m class of models).

$$
\begin{aligned}
R_{T_n,ARI_m}(t) &= P(T_{n+1} - T_n > t|\Im_t) = P(N(T_n, T_n + t] = 0|\Im_t) \\
&= exp\left\{ \left(\frac{t_n}{\eta}\right)^{\beta} - \left(\frac{t_n + t}{\eta}\right)^{\beta} \right\} \times \\
&\times \quad exp\left\{ t(1-\theta) \sum_{j=0}^{Min(m-1,n)-1)} \theta^j \frac{\beta}{\eta} \left(\frac{t_{n-j}}{\eta}\right)^{\beta-1} \right\}.
\end{aligned}
$$

Dump trucks data revisited (cont.)



Final Remarks

- ✓ A Non-Parametric test for MR was proposed against the IR alternative.
- ✓ A extension of the test and a Graphical Technique are still under development.
- ✓ Imperfect Repair Classes of Models (ARA and ARI): Inference, models Selection and Reliability Predictors).
- ✓ Illustration in a real data set related to mining dump trucks.
- ✓ Effect of repair, aging speed and reliability predictor for maintenance policy were estimated for the dump trucks data.

Author's Accepted Manuscript

www.elsevier.com/locate/ress

ARA and ARI imperfect repair models: estimation, goodness-of-fit and reliability prediction

Maria Luíza Guerra de Toledo, Marta A. Freitas, Enrico A. Colosimo, Gustavo L. Gilardoni

PII: S0951-8320(15)00101-5
DOI: http://dx.doi.org/10.1016/j.ress.2015.03.035
Reference: RESS5283

To appear in: *Reliability Engineering and System Safety*

Received date: 12 August 2014
Revised date: 19 February 2015
Accepted date: 28 March 2015

Cite this article as: Maria Luíza Guerra de Toledo, Marta A. Freitas, Enrico A. Colosimo, Gustavo L. Gilardoni, ARA and ARI imperfect repair models: estimation, goodness-of-fit and reliability prediction, *Reliability Engineering and System Safety,* http://dx.doi.org/10.1016/j.ress.2015.03.035

THANK YOU!

MERCI !