Simultaneous assesment of the efficiency of preventive and corrective maintenances for repairable systems in a competing risk framework

Yann DIJOUX, Laurent DOYEN, Olivier GAUDOIN

Institut National Polytechnique de Grenoble
Laboratoire LMC
BP 53 - 38 041 Grenoble Cedex 9 France

All along their life, complex industrial systems are subjected to two kinds of maintenance tasks.

– **Corrective Maintenance (CM, repair)** :
carried out after a failure, intends to put the system into a state in which it can perform its function again.

– **Preventive Maintenance (PM)** :
carried out when the system is in operational conditions, intends to slow down the wear process and reduce the frequency of occurrence of failures.

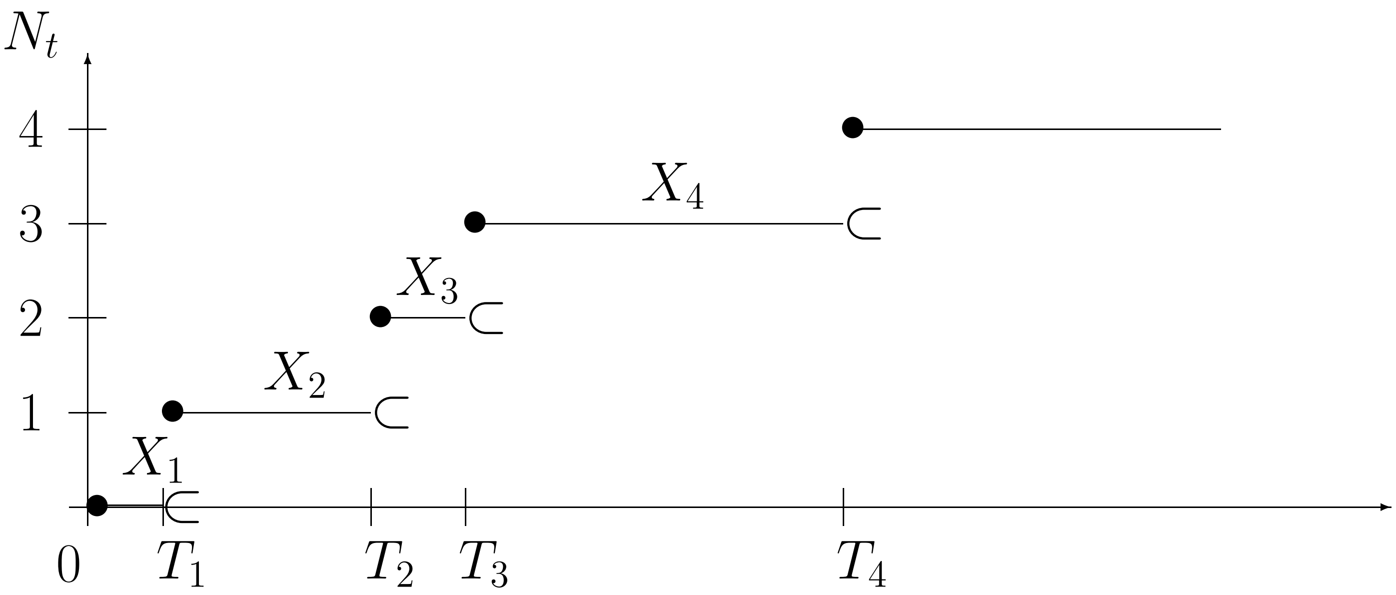
- Planned PM : occur at predetermined times (deterministic PM).
- Condition-based PM : occur at times which are determined according to the results of inspections and degradation or operation controls (random PM).

Aim of the talk : Present a general framework for the stochastic modelling of the maintenance process and the assessment of the efficiency of CM and condition-based PM.

Contents

1. Models with only corrective maintenances

Notations



– failure times : $\{T_i\}_{i\geq 1}$
– inter failure times : $X_i = T_i - T_{i-1},\ i \geq 1$
– counting failure process : $\{N_t\}_{t\geq 0}$, N_t = number of failures occured at time t

- A CM is performed after each failure.
- Repair times are negligible or not taken into account.
- Two failures can not occur at the same time.

Stochastic modelling of the failure process

the failure intensity : λ_t

$$\lambda_t = \lim_{dt \to 0} \frac{1}{dt} P(N_{t+dt} - N_t = 1 | \mathcal{H}_t)$$

where $\mathcal{H}_t$ is the history of the failure process at time t.

Self-excited point process : $\mathcal{H}_t = \sigma\left(\{N_s\}_{0 \le s \le t}\right)$.

$\Rightarrow \lambda_t$ completely defines the failure process.

the likelihood function

The likelihood function for an observation of the failure process with n failures on $[0, t]$:

$$L_t(\theta) = \left[\prod_{i=1}^{n} \lambda_{t_i}(i-1; t_1, \ldots, t_{i-1})\right] exp\left(-\sum_{i=1}^{n} \int_{t_{i-1}}^{t_i} \lambda_u(i-1; t_1, \ldots, t_{i-1})\, du\right)$$

Stochastic modelling of the failure process

the initial failure intensity $\lambda(t)$

Before the first failure, the failure intensity is a deterministic and continuous function of time $\lambda(t)$, the failure rate of T_1.

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} P(t < T_1 \leq t + \Delta t | T_1 > t)$$

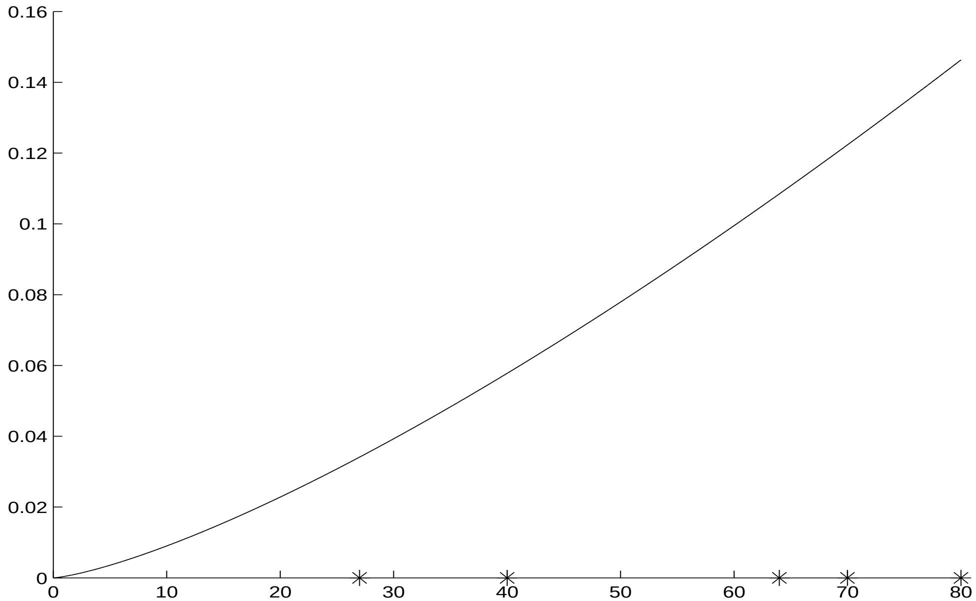
Wearing systems : $\lambda(t)$ is strictly increasing.

example of initial intensity

$$\lambda(t) = \alpha\beta t^{\beta-1} \qquad \alpha > 0 \, , \beta > 0$$

$$\Rightarrow \begin{cases} \beta > 1 & : \text{ wear out (industrial systems).} \\ \beta < 1 & : \text{ improvement (software).} \\ \beta = 1 & : \text{ no ageing (PPH).} \end{cases}$$

Minimal Repair or As Bad As Old model

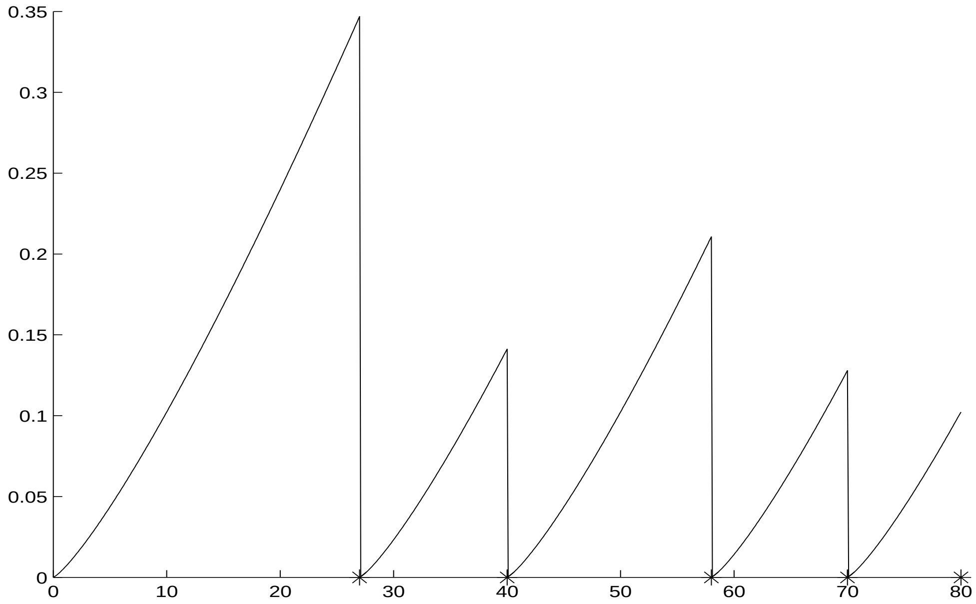


Each maintenance leaves the system in the same state as it was before failure.

Statistical modelling : a Non Homogeneous Poisson Process (NHPP).

$$\lambda_t = \lambda(t)$$

Perfect repair or As Good As New Model AGAN



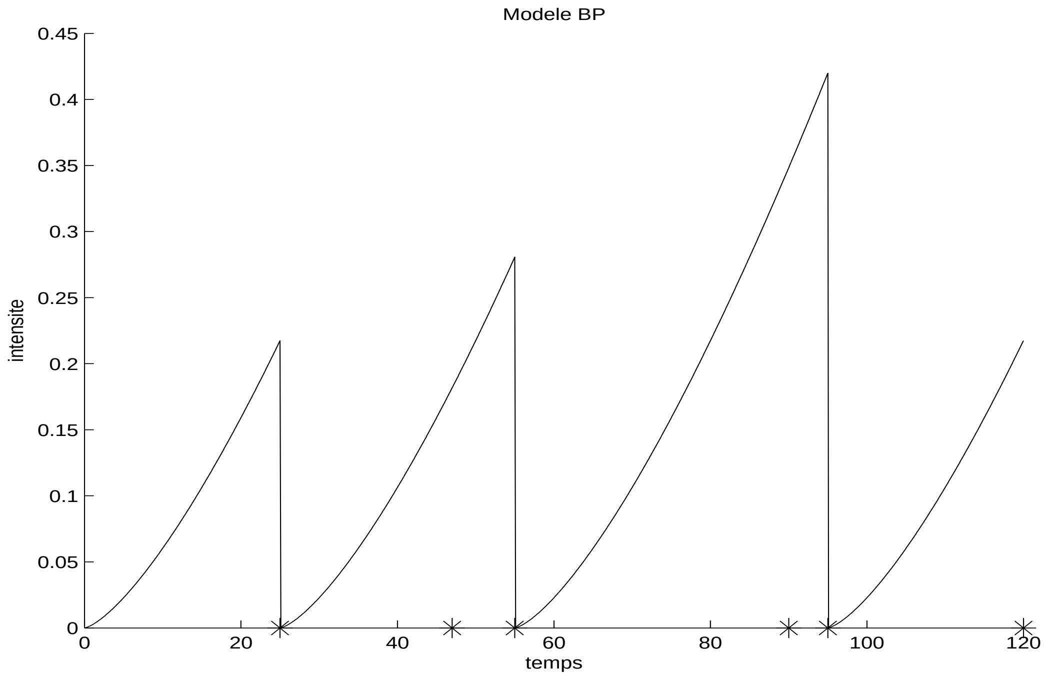
Each maintenance perfectly repairs the system and leaves it as if it were new.

Statistical modelling : the Renewal Process (RP).

$$\lambda_t = \lambda(t - T_{N_t})$$

Reality is between the case ABAO and AGAN

The Brown-Proschan model [1983]



Each maintenance is perfect (AGAN) with probability p and minimal (ABAO) with probability $1 - p$.

Statistical modelling : $\left\{ \begin{array}{ll} B_i = 1 & : i^{\text{th}} \text{ repair AGAN} \\ B_i = 0 & : i^{\text{th}} \text{ repair ABAO} \end{array} \right. , B_i \overset{iid}{\rightsquigarrow} \mathcal{B}(p)$

$$\lambda_t = \lambda(t - T_{N_t} + \sum_{j=1}^{N_t} (\prod_{k=j}^{N_t} (1 - B_k)) X_j)$$

Virtual age models

After the i^{th} repair, the system performs as a new one having survived until A_i.

$$\forall i \geq 0, \quad \forall t \geq 0 P(X_{i+1} > t | X_1, ..X_i) = P(X_1 > A_i + t | X_1 > A_i) = \frac{S(A_i + t)}{S(A_i)}$$

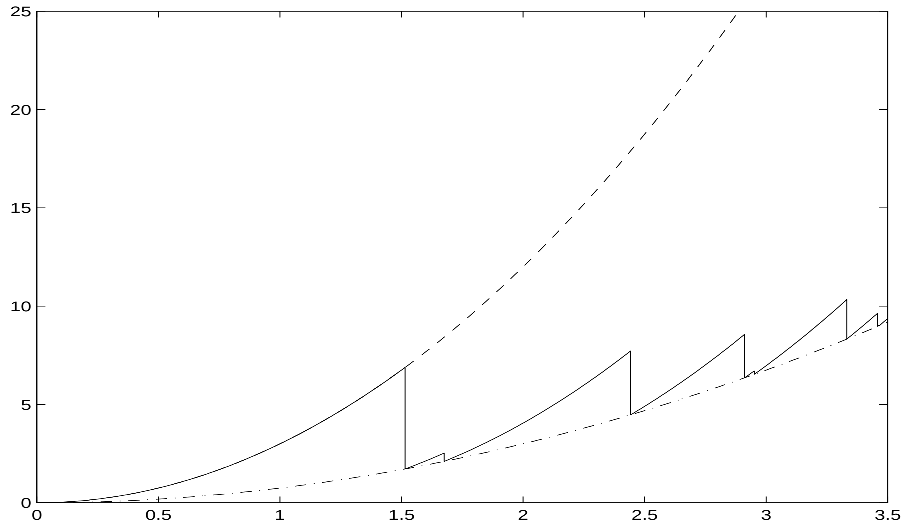
where S is the survival function associated to X_1.

$$\lambda_t = \lambda(t - T_{N_t} + A_{N_t})$$

The A_i are called the **effective ages**. $A_0 = 0$.
Properties : Virtual age models are a generalization of previous models :

- ABAO : $A_i = T_i$
- AGAN : $A_i = 0$
- BP : $A_i = \sum_{j=1}^{i} (\prod_{k=j}^{i} (1 - B_k)) X_j) =$ time elapsed since last perfect repair

The Arithmetic Reduction of Age model ARA_1



After a repair, the system have an age age proportional to his true age :

$$A_i = (1 - \rho)T_i$$

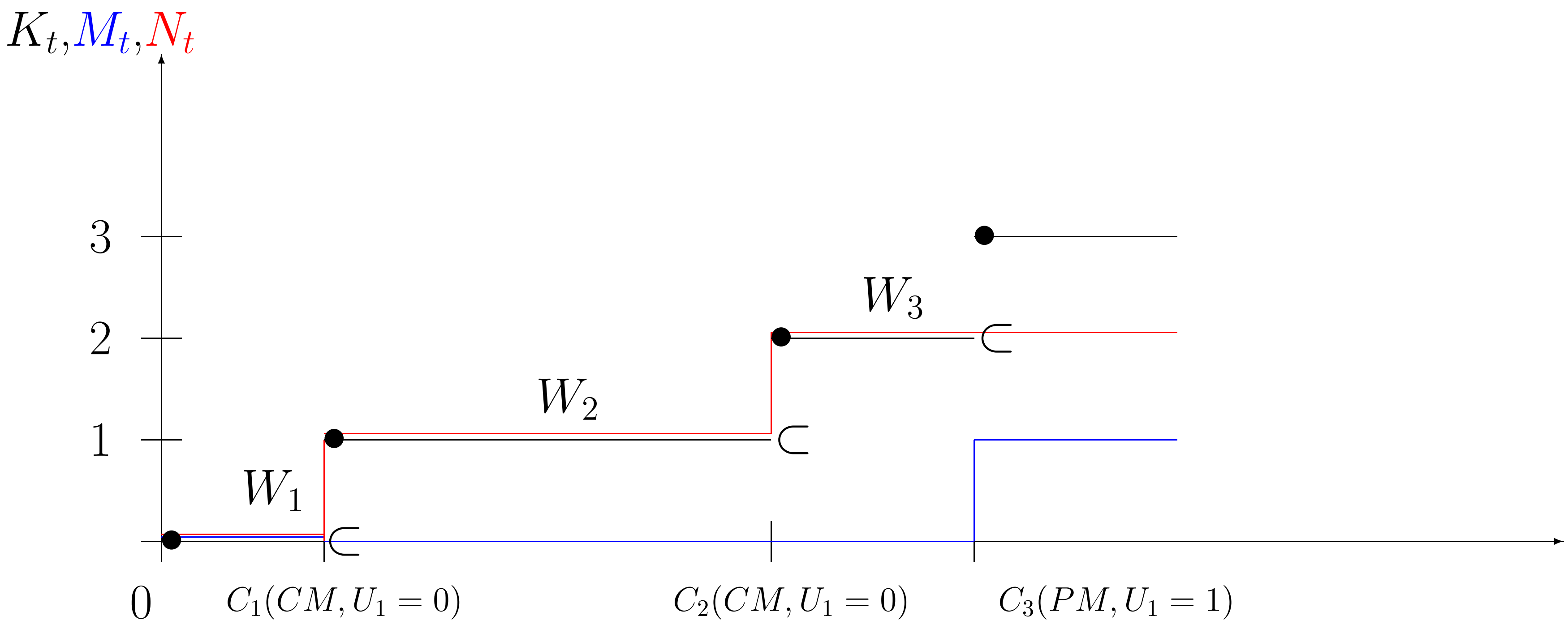
ρ is called the **improvement factor** or **repair efficiency** :

$$\rho = 0 \Rightarrow \text{ABAO}$$
$$\rho = 1 \Rightarrow \text{AGAN}$$

$$\lambda_t = \lambda(t - \rho T_{N_t})$$

2. Models with two kind of maintenances

Notations



– maintenances times (CM+PM) : $\{C_i\}_{i\geq 1}$

– inter-maintenance times (CM+PM) : $W_i = C_i - C_{i-1},\ i \geq 1$

– The counting maintenance process : $\begin{cases} \{K_t\}_{t\geq 0} & \text{PM+CM} \\ \{N_t\}_{t\geq 0} & \text{CM} \\ \{M_t\}_{t\geq 0} & \text{PM} \end{cases}$

– the types of maintenances : $U_i = \begin{cases} 1 & \text{if the } i^{\text{th}} \text{ maintenance is preventive} \\ 0 & \text{if the } i^{\text{th}} \text{ maintenance is corrective} \end{cases}$

Stochastic modelling of the global process

The maintenance intensities :

– The global maintenance intensity : $\lambda_t^K(K,U) = \lim\limits_{dt\to 0} \dfrac{1}{dt} P(K_{t+dt} - K_t = 1|\mathcal{H}_t)$

– The CM intensity : $\lambda_t^N(K,U) = \lim\limits_{dt\to 0} \dfrac{1}{dt} P(N_{t+dt} - N_t = 1|\mathcal{H}_t)$

– The PM intensity : $\lambda_t^M(K,U) = \lim\limits_{dt\to 0} \dfrac{1}{dt} P(M_{t+dt} - M_t = 1|\mathcal{H}_t)$

where $\mathcal{H}_t$ is the history of the maintenance process at time t.

Typically, $\mathcal{H}_t = \sigma\left(\{K_s, U_{K_s}\}_{0\leq s\leq t}\right)$.

– $\lambda_t^K(K,U) = \lambda_t^N(K,U) + \lambda_t^M(K,U)$

– The PM and CM intensities completely define the maintenance process.

Main results

Jacod's formulae : $P(W_{k+1} > w, U_{k+1} = 0 | W_1 = w_1, \ldots, U_k = u_k) =$

$$\int_w^{+\infty} \lambda^N_{c_k+u}(k, w_1, \ldots, u_k) P(W_{k+1} > u | w_1, \ldots, u_k) du$$

$$P(W_{k+1} > w | W_1 = w_1, \ldots, U_k = u_k) = exp\left(-\int_0^w \lambda^K_{c_k+s}(k, w_1, \ldots, u_k)\, ds\right)$$

$$P(U_{k+1} = 0 | W_{k+1} = w, W_1 = w_1, \ldots, U_k = u_k) = \frac{\lambda^N_{c_k+w}(k, w_1, \ldots, u_k)}{\lambda^K_{c_k+w}(k, w_1, \ldots, u_k)}$$

The likelihood function associated to an observation of the PM-CM process with k maintenances on $[0, t]$:

$$L_t(\theta) = exp\left(-\sum_{j=1}^{k+1} \int_{c_{j-1}}^{c_j} \lambda^K_s(j-1, w_1, \ldots, u_{j-1})\, ds\right)$$

$$\left[\prod_{i=1}^{k} \lambda^N_{c_i}(i-1, w_1, \ldots, u_{i-1})^{1-u_i}\, \lambda^M_{c_i}(i-1, w_1, \ldots, u_{i-1})^{u_i}\right] \quad \text{where } c_0 = 0, \quad c_{k+1} = t.$$

Classical competing risk models

Cooke-Bedford (2002) : Competing risks (CR) approach for the PM-CM process.

After the $(k-1)^{th}$ maintenance, the **risk variables** are :

- Y_k = potential time to the next PM if no CM occur before (risk of PM)
- Z_k = potential time to the next CM if no PM occur before (risk of CM)

Observations

In practice, Y_i and Z_i are not observed. The observations are :

- The time to next maintenance : $W_i = \min(Y_i, Z_i)$
- The type of next maintenance : $U_i = \begin{cases} 1 & \text{if} \quad Y_i < Z_i \quad \text{(PM)} \\ 0 & \text{if} \quad Z_i < Y_i \quad \text{(CM)} \end{cases}$

Main aspects of the classical competing risk model

Examples of CR models :

- Independent competing risks : $Y_i \perp Z_i$
  Drawback : non realistic model.
- Random time censoring :

$$Z_i \text{ independent from } sign(Z_i - Y_i) \Rightarrow Z_i \text{ independent from } U_i$$

Main drawbacks of the classical approach :

- By definition, the maintenance are supposed AGAN.
- The joint distribution $S_1(y, z) = P(Y_1 > y, Z_1 > z)$is generally not identifiable. Indeed, we can estimate the sub-survival functions :

$$S^*_{Z_1}(z) = P(Z_1 > z, Z_1 < Y_1) = P(W_1 > z, U_1 = 0) \quad (1)$$

$$S^*_{Y_1}(y) = P(Y_1 > y, Y_1 \leq Z_1) = P(W_1 > y, U_1 = 1) \quad (2)$$

Idea : Define identifiable models taking into account of the past of the process in order to estimate a maintenance effect.

Generalized competing risk models

PM-CM (conditional) survival function :

$$S_{k+1}(y, z; w_1, \dots, u_k) = P(Y_{k+1} > y, Z_{k+1} > z | W_1 = w_1, \dots, U_k = u_k)$$

Generalized sub-survival functions :

$$\begin{aligned} S^*_{Z_{k+1}}(z; w_1, \dots, u_k) &= P(Z_{k+1} > z, Z_{k+1} < Y_{k+1} \,|\, W_1 = w_1, \dots, U_k = u_k) \\ &= \int_z^{+\infty} \left[-\frac{\partial}{\partial z} S_{k+1}(y, z; w_1, \dots, u_k) \right]_{(s,s)} ds \end{aligned}$$

$$\begin{aligned} S^*_{Y_{k+1}}(y; w_1, \dots, u_k) &= P(Y_{k+1} > y, Y_{k+1} < Z_{k+1} \,|\, W_1 = w_1, \dots, U_k = u_k) \\ &= \int_y^{+\infty} \left[-\frac{\partial}{\partial y} S_{k+1}(y, z; w_1, \dots, u_k) \right]_{(s,s)} ds \end{aligned}$$

Generalized competing risk models (2)

Link with the colored point process approach :

$$
\begin{aligned}
S_{k+1}(w, w; w_1, \ldots, u_k) &= P(W_{k+1} > w | w_1, \ldots, u_k) \\
S^*_{Z_{k+1}}(w; w_1, \ldots, u_k) &= P(W_{k+1} > w, U_{k+1} = 0 | w_1, \ldots, u_k) \\
S^*_{Y_{k+1}}(w; w_1, \ldots, u_k) &= P(W_{k+1} > w, U_{k+1} = 1 | w_1, \ldots, u_k)
\end{aligned}
$$

$$
\begin{aligned}
\lambda_t^N(K, U) &= \frac{\left[-\frac{\partial}{\partial z} S_{K_t+1}(y, z; w_1, \ldots, u_{K_t})\right]_{(t-c_{K_t}, t-c_{K_t})}}{S_{K_t+1}(t - c_{K_t}, t - c_{K_t}; w_1, \ldots, u_{K_t})} \\
\lambda_t^M(K, U) &= \frac{\left[-\frac{\partial}{\partial y} S_{K_t+1}(y, z; w_1, \ldots, u_{K_t})\right]_{(t-c_{K_t}, t-c_{K_t})}}{S_{K_t+1}(t - c_{K_t}, t - c_{K_t}; w_1, \ldots, u_{K_t})} \\
\lambda_t^K(K, U) &= -\frac{d}{dt} \ln S_{K_t+1}(t - c_{K_t}, t - c_{K_t}; w_1, \ldots, u_{K_t})
\end{aligned}
$$

Generalized competing risk models (3)

Ex : **Conditionnally independent generalized competing risks**

$\forall k, Y_{k+1}$ and Z_{k+1} are independent conditionnally to $W_1, \ldots, U_k$.

The intensities depend only on the values of the PM-CM survival function around the first diagonal

$\Rightarrow$ same **identifiability problem** as in classical competing risks : for any GCR model, there exists a conditionnally independent GCR model with the same PM and CM intensities.

Likelihood :

$$L_t(\theta) = S_{k+1}(t - c_k, t - c_k; w_1, \ldots, u_k)$$

$$\left[\prod_{i=1}^{k} \left[-\frac{\partial}{\partial y} S_i(y, z; w_1, \ldots, u_{i-1}) \right]_{(w_i, w_i)}^{u_i} \left[-\frac{\partial}{\partial z} S_i(y, z; w_1, \ldots, u_{i-1}) \right]_{(w_i, w_i)}^{1-u_i} \right]$$

Generalized virtual age models

Idea of the model : there exist a sequence of effective ages $\{A_k\}_{k\geq 1}$, with $A_0 = 0$, such that after k^{th} maintenance, the risk variables Y_{k+1} and Z_{k+1} behave as the risk variables of a new system with no maintenance before A_k :

$$P(Y_{k+1} > y, Z_{k+1} > z \,|\, w_1, \ldots, u_k, A_k) = P(Y > A_k+y, Z > A_k+z | Y > A_k, Z > A_k, A_k)$$

where (Y, Z) is a random couple with the same distribution as (Y_1, Z_1).

The effect of maintenance is symmetrical on both risks.

PM-CM Survival function : $S_{k+1}(y, z; w_1, \ldots, u_k) = \dfrac{S_1(A_k + y, A_k + z)}{S_1(A_k, A_k)}$

Virtual age property on the times between maintenances :

$$P(W_{k+1} > w | w_1, \ldots, u_k, A_k) = P(W_1 > w + A_k | W_1 > A_k, A_k)$$

Main results in the virtual ages approach

The complete maintenance intensities

$$\lambda_t^N(K,U) = \lambda_c(t - C_{K_t} + A_{K_t}) \quad (3)$$
$$\lambda_t^M(K,U) = \lambda_p(t - C_{K_t} + A_{K_t}) \quad (4)$$
$$\lambda_t^K(K,U) = \lambda(t - C_{K_t} + A_{K_t}) \quad (5)$$

where :

$$\lambda_c(t) = \frac{\left[-\frac{\partial}{\partial z} S_1(y,z)\right]_{(t,t)}}{S_1(t,t)}; \quad \lambda_p(t) = \frac{\left[-\frac{\partial}{\partial y} S_1(y,z)\right]_{(t,t)}}{S_1(t,t)}; \quad \lambda(t) = \lambda_c(t) + \lambda_p(t)$$

The generalized sub-survival functions

$$S^*_{Z_{k+1}}(z; w_1, \dots, u_k) = \int_z^{+\infty} \lambda_c(s + A_k)\, exp\left(-\int_0^s \lambda(u + A_k) du\right) ds \quad (6)$$

$$S^*_{Y_{k+1}}(y; w_1, \dots, u_k) = \int_y^{+\infty} \lambda_p(s + A_k)\, exp\left(-\int_0^s \lambda(u + A_k) du\right) ds \quad (7)$$

How to build a generalized virtual age model

1. Define the dependency between both kind of maintenances by characterizing the joint survival function S_1
2. Define the effective ages by characterizing the both maintenances effects.
3. Derive λ_c and λ_p from S_1.

Assumptions on maintenance efficiency :

– PM and CM AGAN : each maintenance restores the system as new (RP) : $A_k = C_k$

$$\lambda_t^N(K,U) = \lambda_c(t - C_{K_t})$$

– PM and CM ABAO : each maintenance is minimal (NHPP) : $A_k = C_k$

$$\lambda_t^N(K,U) = \lambda_c(t)$$

– PM and CM BP

$$\begin{cases} B_i = 1 : i^{\text{th}} \text{ maintenance AGAN} \\ B_i = 0 : i^{\text{th}} \text{ maintenance ABAO} \end{cases} \& \begin{cases} B_i \rightsquigarrow \mathcal{B}(p_p) \text{ if } U_i = 1 \\ B_i \rightsquigarrow \mathcal{B}(p_c) \text{ if } U_i = 0 \end{cases}$$

$$\lambda_t^N(K,U) = \lambda_c\left(t - C_{K_t} + \sum_{j=1}^{K_t}\sum_{k=j}^{K_t}(1 - B_k)W_k\right) \quad (8)$$

Generalized ARA_1 model

The effect of PM is to reduce the virtual age of ρ_p times the time elapsed since last maintenance, and the effect of CM is similar with a different parameter ρ_c :

$$A_k = \begin{cases} A_{k-1} + W_k - \rho_p W_k & \text{if } U_k = 1 \\ A_{k-1} + W_k - \rho_c W_k & \text{if } U_k = 0 \end{cases} \Rightarrow A_k = C_k - \sum_{i=1}^{k} \rho_p^{U_i} \rho_c^{1-U_i} W_i$$

$$\lambda_t^N(K,U) = \lambda_c \left(t - \sum_{i=1}^{K_t} \rho_p^{U_i} \rho_c^{1-U_i} W_i \right)$$

- $\rho_p = 0 \Rightarrow$ PM are ABAO. $\rho_c = 0 \Rightarrow$ CM are ABAO.
- $\rho_p = 1 \Rightarrow$ PM are not AGAN but "As Good As Previous" : PM restores the system in the state it was just after previous CM ($A_k = A_{k-1}$, not 0). Then, PM cannot prevent the ageing due to CM.
- $\rho_p = 1$ and $\rho_c = 1 \Rightarrow$ PM and CM are AGAN.

Dependency between PM and CM (1)

Independent Risks Model : Z_1 and Y_1 are independent

$\Rightarrow \lambda_c$ and λ_p are respectively the hazard rates of Z_1 and Y_1

$$\lambda_c(t) = \frac{-S'_{Z_1}(t)}{S_{Z_1}(t)} \qquad \lambda_p(t) = \frac{-S'_{Y_1}(t)}{S_{Y_1}(t)}$$

Example with Weibull distributions for Z_1 and Y_1 :

$$\lambda_t^N(K,U) = \alpha_c \beta_c (t - C_{K_t} + A_{K_t})^{\beta_c - 1} \qquad \lambda_t^M(K,U) = \alpha_p \beta_p (t - C_{K_t} + A_{K_t})^{\beta_p - 1}$$

Not realistic for condition-based PM.

Dependency between PM and CM (2)

A non independent Risks Model : the maintenance type U_1 is independent of the maintenance time W_1.

$$\Rightarrow \qquad S^*_{Z_1}(z) = (1-q)S_1(y,y) \qquad \text{and} \qquad S^*_{Y_1}(y) = qS_1(y,y)$$

where $q = P(U_1 = 1)$.

$$\lambda_c(t) = (1-q)\lambda_{W_1}(t) \qquad \lambda_p(t) = q\lambda_{W_1}(t)$$

Example with Weibull distributions for W_1 :

$$\lambda_t^N(K,U) = (1-q)\alpha\beta(t - C_{K_t} + A_{K_t})^{\beta-1} \qquad \lambda_t^M(K,U) = q\alpha\beta(t - C_{K_t} + A_{K_t})^{\beta-1}$$

Very simple model.

The Langseth & Lindqvist model [LL,2003]

LL assumptions :

1. Random sign model : $U_1 \perp Z_1$
2. PM and CM are of the BP type.
3. The hazard rate of Z_1 is $\lambda(t)$.
4. The link between the two kind of risks Y_1 and Z_1 are defined as follows :
$$P(Y_1 \leq y | Z_1 = z, Y_1 < Z_1) = \frac{\Lambda(y)}{\Lambda(z)} \ , \Lambda(t) = \int_0^t \lambda(x) dx$$
This hypothesis allows to perform PM just before a CM occurs.
5. $q = P(Y_1 < Z_1) = P(U_1 = 1)$

Problem : In practice, the maintenance effects (B_k) are not observed.

$\Rightarrow$ the likelihood function has a recursive expression, difficult to use.

$\Rightarrow$ we prefer using another virtual age model, e.g. ARA_1.

Main results of the LL model with ARA_1 assumptions

The survival and sub-survival functions :

- $S_Y^*(y) = P(W_1 > y, U_1 = 1) = q(e^{-\Lambda(y)} - \Lambda(y) Ie(\Lambda(y)))$
- $S_Z^*(z) = P(W_1 > z, U_1 = 0) = (1-q)e^{-\Lambda(z)}$
- $S(t) = P(W_1 > t) = e^{-\Lambda(t)} - q\Lambda(t) Ie(\Lambda(t))$

The initial maintenance intensities :

$$\lambda_c(t) = \frac{(1-q)\, \lambda(t)\, exp\,(-\Lambda(t))}{exp\,(-\Lambda(t)) - q\, \Lambda(t)\, Ie(\Lambda(t))} \tag{9}$$

$$\lambda_p(t) = \frac{q\, \lambda(t)\, Ie(\Lambda(t))}{exp\,(-\Lambda(t)) - q\, \Lambda(t)\, Ie(\Lambda(t))} \tag{10}$$

with $Ie(t) = \int_t^{+\infty} e^{-s}/s\, ds$

The likelihood function :

$$\mathcal{L}_t = [\prod_{i=1}^{n} \frac{\lambda(y_{c_i}^{i-1})(1-q)^{1-u_i} q^{u_i} e^{-\Lambda(y_{c_i}^{i-1})(1-u_i)} Ie^{u_i}(\Lambda(y_{c_i}^{i-1}))}{e^{-\Lambda(y_{c_{i-1}}^{i-1})} - q\Lambda(y_{c_{i-1}}^{i-1}) Ie(\Lambda(y_{c_{i-1}}^{i-1}))}] * \frac{e^{-\Lambda(y_t^n)} - q\Lambda(y_t^n) Ie(\Lambda(y_t^n))}{e^{-\Lambda(y_{c_n}^n)} - q\Lambda(y_{c_n}^n) Ie(\Lambda(y_{c_n}^n))}$$

with $y_s^k = s - \sum_{j=1}^{k} \rho_p^{u_i} \rho_c^{1-u_i} w_j$

3. An application to real data

Data and assumptions

Data :

– Data of specific systems used in power plants issued from the French Electricity company EDF.
– 17 similar and independent production units with times of maintenances and right censored.
– 15 years of observation, 16 PM and 12 CM observed.

Assumptions of the model :

– The LL model with ARA maintenance effects.
– Z_1 has a Weibull distribution :

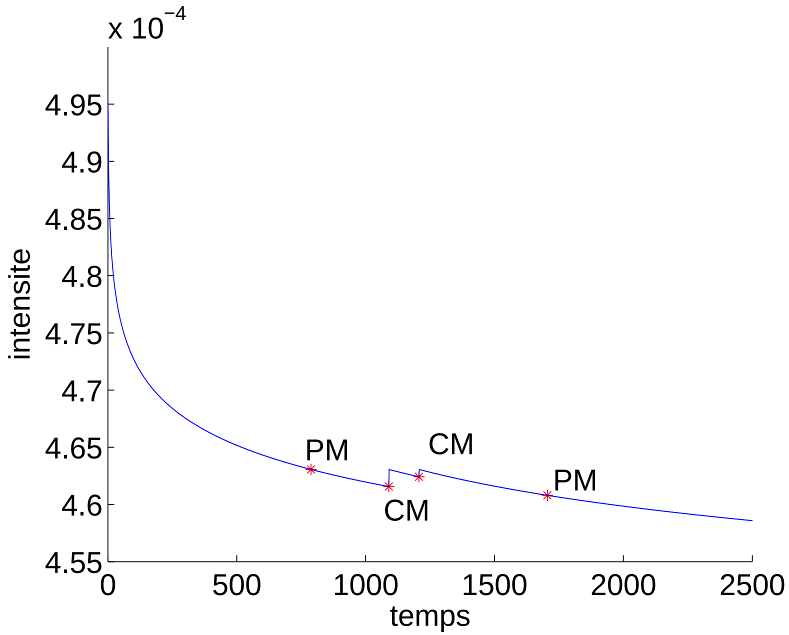
$$\lambda(t) = \alpha\beta t^{\beta-1}$$

– the 17 systems are *iid*.

$$\Rightarrow \boxed{\text{Estimate } \; \theta = (\alpha, \beta, \rho_p, \rho_c, q)}$$

Estimation of the parameters

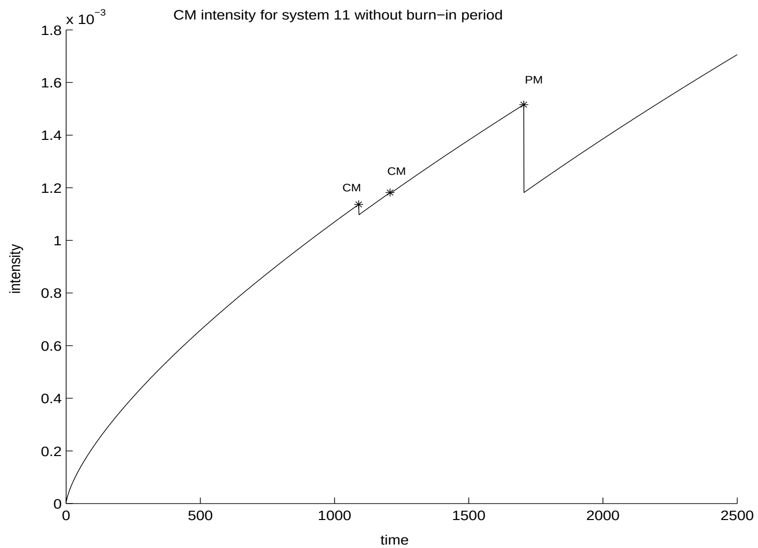
$$
\begin{array}{rcccc}
\hat{\alpha} & = & 0.00045 & & \\
\hat{\beta} & = & 0.95 & \Rightarrow & \text{global improvement.} \\
\hat{\rho}_c & = & 0.95 & \Rightarrow & \text{harmful CM.} \\
\hat{\rho}_p & = & 0.01 & \Rightarrow & \text{minimal PM (ABAO).} \\
\hat{q} & = & 0.55 & \Rightarrow & \hat{q} \approx \text{proportion of PM (57 \%.)}
\end{array}
$$



$\Rightarrow$ Remove the burn-in period (3 years) : 10 PM and 9 CM remain.

Estimation of the parameters without the burn-in period

$$\begin{array}{rclcl}
\hat{\alpha} & = & 5\ 10^{-7} & & \\
\hat{\beta} & = & 1.70 & \Rightarrow & \text{global ageing.} \\
\hat{\rho}_0 & = & 0.03 & \Rightarrow & \text{nearly minimal CM.} \\
\hat{\rho}_1 & = & 0.99 & \Rightarrow & \text{PM nearly optimal but not AGAN} \\
\hat{q} & = & 0.47 & \Rightarrow & \hat{q} \approx \text{proportion of PM}(52\ \%.)
\end{array}$$



Conclusions and Prospects

Conclusions

- General modelling of the effect of PM and CM, with possibly dependent PM and CM times
- Simultaneous estimation of parameters linked to the wear-out process and maintenance efficiency
- Great help for the monitoring of the reliability centered maintenance process

Prospects

- Take into account the burn-in period of the systems :
  - Add a risk variable specific to this period.
  - Choose fitted failure intensities like bathtub shaped intensities.
  - Adapt the maintenances effects to burn-in period.
- Change the dependency between CM and PM.
- Study the conditionnally independent generalized competing risks.

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