A generalized Brown-Proschan model for preventive and corrective maintenance

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I. Introduction

The dependability of complex repairable systems depends strongly on the efficiency of maintenance actions.

Corrective Maintenance (CM): After a failure, put the system into a state in which it can perform its function again.

Preventive Maintenance (PM): When the system is operating, intends to slow down the wear process.

Basic maintenance effect assumptions:

- As Bad As Old (ABAO): restores the system in the same state it was just before maintenance.
- As Good As New (AGAN): restores the system as if it were new.

Reality is between these two extreme cases: **Imperfect maintenance**.

Brown-Proschan model [83], CM is :

- **AGAN** with probability **p**,
- **ABAO** with probability 1 p.
- repair effects (AGAN or ABAO) are mutually independent and independent of already observed failure times.

CM effects can be characterized with random variables:

 $\boldsymbol{B_{i}} = \begin{cases} 1 & \text{if the } i^{\text{th}} \text{ CM is AGAN} \\ 0 & \text{if the } i^{\text{th}} \text{ CM is ABAO} \end{cases}$

Statistical studies when the $\{B_i\}_{i\geq 1}$ are known: Whitaker-Samaniego[89], Hollander-Presnell-Sethuraman [92], Kvam-Singh-Whitaker [02], Bathe-Franz [96], Agustin-Pena[99],...

Practical purposes : $\{B_i\}_{i\geq 0}$ are hidden variables.

- p = 0 : ABAO
- p characterizes maintenance efficiency: p = 1 : AGAN
 - 0 : imperfect

Joint assessment of CM efficiency and intrinsic wear-out: Lim [98]; Lim-Lie[00]; Lim-Lu-Park[98]; Langseth-Lindqvist [04].

Presentation aim:

- Generalize the BP model to PM effects
- Assess PM efficiency and intrinsic wear-out when PM effects are unknown
- Compute reliability indicators.

II/ A maintenance data set

PM and CM times of a subsystem of a fossil-fired thermal plant of EDF:

1179	2640	4101	5562	7023	8035	8329	8376	8393	8455
8484	8494	8605	8628	8641	8744	8903	9105	9660	9845
9846	9866	9919	9985	9987	10010	10363	10470	11021	11494
12851	12956	13662	14161	14217	14244				



III. Notations



IV. Model assumptions

- $\lambda(t)$ the failure rate of the new unmaintained system. $\Lambda(t) = \int_0^t \lambda(s) \, ds$,
- Maintenance durations are not taken into account,
- PM are done at deterministic times, all the PM times are known
- CM are done at random times, CM are observed over [c, T] (c < T)
- **CM** effects are **ABAO**,
- \bullet \mathbf{PM} effects follow a Brown-Proschan model, i.e. PM effects are :
 - mutually independent and independent of previous failure times,
 - AGAN with probability p, $B_i = \begin{cases} 1 & \text{if the } i^{\text{th}} \text{ PM is AGAN} \end{cases}$
 - **ABAO** with probability 1 p.

$$\mathbf{e}_{i} = \begin{cases} 1 & \text{if the } i & \text{PM is AGA} \\ 0 & \text{if the } i^{\text{th}} & \text{PM is ABA} \end{cases}$$

$$P(B_i = 1 \mid \underline{T}_i, \underline{B}_{i-1}) = P(B_i = 1) = p$$

V. Joint assessment of intrinsic wear-out and PM efficiency *a/ Maximum likelihood method*

Likelihood associated to a single observation of the failure process over [c, t]:

$$L_t(\theta) = f_{\underline{T}_{n_t} \mid N_t = n_t}(\underline{t}_{n_t}) P(N_t = n_t)$$

Maximum Likelihood Estimator (MLE):

$$\hat{\theta} = \arg \max_{\theta} L_T(\theta) = \arg \max_{\theta} log(L_T(\theta))$$

Notations: $L_c(\theta) = 1$

 $L_{\tau,t}(\theta) = f_{\underline{T}_{n_t-n_\tau}, N_{t-\tau}=n_t-n_\tau}(t_{n_\tau+1}-\tau, ..., t_{n_t}-\tau) \Rightarrow$ the likelihood associated to the system new in τ and observed over $[c \lor \tau, t]$.

Let us denote :
$$D_t^m = (1-p)^{m_t - m} \left[\prod_{c \lor \tau_m \le t_j \le t} \lambda(t_j - \tau_m) \right]$$

Two equivalent equations for likelihood recursive computation:

•
$$L_t(\theta) = \left[\sum_{m=0}^{m_t-} p^{\mathbbm{1}_{\{m\neq 0\}}} D_t^m e^{-(\Lambda(t-\tau_m)-\Lambda((c\vee\tau_m)-\tau_m))} L_{c\vee\tau_m}(\theta)\right]$$

 \Rightarrow forward computation algorithm.

•
$$L_t(\theta) = \left[\sum_{\tau \in \{\tau_1, \dots, \tau_{m_t}, t\}} p^{\mathbb{1}_{\{\tau \neq t\}}} D^0_{\tau_m} e^{-(\Lambda(\tau_m \lor c) - \Lambda(c))} L_{\tau_m, t}(\theta) \right]$$

 \Rightarrow backward computation algorithm.

Proof of the equation used for the forward computation:

 $\underbrace{\{B_m = 1, \{B_j = 0\}_{m < j \le m_t -}\}_{0 \le m \le m_t -}}_{\tau_m \text{ is the last AGAN PM since } t} \mathbb{O}_{m \le m_t - t} \text{ is a partition of the probability space.}$

Given $B_m = 1$, the CM process can be divided into 2 independent processes.

$$L_{t}(\theta) = \left[\sum_{m=0}^{m_{t}-} \underbrace{p^{\mathbb{I}_{\{m\neq0\}}}(1-p)^{m_{t}--m}}_{P(B_{m}=1,\{B_{j}=0\}_{m
Given $B_{m}=1,\{B_{j}=0\}_{m CM times after τ_{m} follow a NHPP initialized in τ_{m}
 $\underbrace{L_{c\vee\tau_{m}}(\theta)}_{Given B_{m}=1, \text{ maintenances before }\tau_{m} \text{ follow a BP PM-ABAO CM model}}$$$$

b/ MLE combined with moment estimation : $\lambda(t) = \alpha \beta t^{\beta-1}$

Power Law Process: $E[N_T] = \alpha(T^{\beta} - c^{\beta}) \Rightarrow \tilde{\alpha}_{\beta} = N_T/(T^{\beta} - c^{\beta}).$

BP model, $E[N_t] = \alpha S_t$, where $S_t = \sum_{\tau \in \{\tau_{m_c+1}, \dots, \tau_{m_t-}, t\}} \sum_{k=0}^{m_{\tau^-}} p^{\mathbb{1}_{\{k \neq 0\}}} (1-p)^{m_{\tau^-}-k} ((\tau - \tau_k)^{\beta} - ((\tau_{m_{\tau^-}} \lor c) - \tau_k)^{\beta})$

MLE combined with moment estimation of α : $\tilde{\alpha}_{\beta,p} = \frac{N_T}{S_T}$

$$E[\tilde{\alpha}_{\beta,p}] = \alpha$$

$$(\tilde{\beta}, \tilde{p}) = \arg \max_{(\beta,p)} \log(L_T(\tilde{\alpha}_{\beta,p}, \beta, p)) \quad \text{and} \quad \tilde{\alpha} = \tilde{\alpha}_{\tilde{\beta}, \tilde{p}}$$

 \Rightarrow Dimension reduction of the likelihood maximization space.

c/ Individual PM efficiency assessment

p characterized the **average global PM efficiency**.

The m^{th} **PM effect** can be characterized by: $\pi_m^{\theta} = P(B_m = 1 \mid N_T = n_T, \underline{T}_{n_T} = \underline{t}_{n_T})$

which verifies:

$$\pi_m^{\theta} = p \frac{L_{c \vee \tau_m}(\theta) \ L_{\tau_m, T}(\theta)}{L_T(\theta)}$$

It can naturally be estimated by: π_m^{θ}

- $L_{\tau_m}(\hat{\theta})$: intermediate computing values of the forward algorithm
- $L_{\tau_m,T}(\hat{\theta})$: intermediate computing values of the backward algorithm

$$L_t^c(\theta) = f_{\underline{T}_{n_t} \mid N_t = n_t, \underline{B}_{n_t} = \underline{b}_{n_t}}(\underline{t}_{n_t}) P(N_t = n_t | \underline{B}_{n_t} = \underline{b}_{n_t}) P(\underline{B}_{n_t} = \underline{b}_{n_t})$$

EM algorithm:

- Expectation (E) step: Compute $Q(\theta|\theta_k) = E^{\theta_k} [log(L_t^c(\theta)) | N]$
- Maximization (M) step: $\theta_{k+1} = \arg \max_{\theta} Q(\theta | \theta_k)$

$$\theta_k \underset{k \to \infty}{\longrightarrow}$$
local likelihood maxima

$$L_t^c(\theta) = \left[\prod_{n=1}^{N_t} \lambda(A_{t_n})\right] e^{-\int_c^t \lambda(A_s) ds} \left[\prod_{m=1}^{m_t} p^{B_m} (1-p)^{1-B_m}\right]$$

where A_s is the virtual age, ie the time elapsed since the last perfect PM $\forall s \in]\tau_m, \tau_{m+1}] \cap [c, T] \quad \lambda(A_s) = \prod_{i=0}^m [\lambda(s - \tau_{m-i})]^{1_{\{B_{\tau_m}^{-i}\}}}$ where $B_{\tau_m}^{-i} =$ "at time τ_m , the last AGAN PM time is τ_{m-i} "

where A_s is the virtual age, ie the time elapsed since the last perfect PM $\forall s \in [\tau_m, \tau_{m+1}] \cap [c, T] \quad \lambda(A_s) = \prod_{i=0}^m [\lambda(s - \tau_{m-i})]^{\mathbb{1}_{\{B_{\tau_m}^{-i}\}}}$ where $B_{\tau_m}^{-i} =$ "at time τ_m , the last AGAN PM time is τ_{m-i} "

$$\underbrace{E^{\theta}[log(L_{t}^{c}(\theta))|N]=Q(p|\theta)}_{L_{t}^{c}(\theta)} = \underbrace{\left[\prod_{n=1}^{N_{t}}\lambda(A_{t_{n}})\right]e^{-\int_{c}^{t}\lambda(A_{s})\,ds}}_{E^{\theta}[log(.)|N]=Q_{2}(\lambda|\theta)} \underbrace{\left[\prod_{m=1}^{m_{t}}p^{B_{m}}(1-p)^{1-B_{m}}\right]}_{E^{\theta}[log(.)|N]=Q_{1}(p|\theta)}$$

where A_s is the virtual age, ie the time elapsed since the last perfect PM $\forall s \in [\tau_m, \tau_{m+1}] \cap [c, T] \quad \lambda(A_s) = \prod_{i=0}^m [\lambda(s - \tau_{m-i})]^{1_{\{B_{\tau_m}^{-i}\}}}$ where $B_{\tau_m}^{-i} =$ "at time τ_m , the last AGAN PM time is τ_{m-i} "

•
$$Q_2$$
 function of $\pi_{m,i}^{\theta} = E^{\theta} \left[\mathbb{1}_{\{B_{\tau_m}^{-i}\}} \mid N_T = n_T, \underline{T}_{n_T} = \underline{t}_{n_T} \right]$
• Q_1 function of $\pi_{m,0}^{\theta} = \pi_m^{\theta} = E^{\theta} \left[B_m \mid N_T = n_T, \underline{T}_{n_T} = \underline{t}_{n_T} \right]$

E step: Compute for $0 \le m \le m_T$ and $0 \le i \le m$,

$$\pi^{ heta}_{m,i} = E^{ heta} \left[\mathbbm{1}_{\{B_{ au_m}^{-i}\}} \mid N_T = n_T, \underline{T}_{n_T} = \underline{t}_{n_T}
ight]$$

• $\pi_{0,0}(\theta) = 1$

• for
$$m \in \{1, ..., m_T\}, \ \pi^{\theta}_{m,0} = \pi^{\theta}_m$$

• for
$$0 \le m < m_T$$
 and $0 \le i \le m$,

$$\pi_{m+1,i+1}^{\theta} = \pi_{m,i}^{\theta} - p^{1+1}_{\{m>i\}}(1-p)^{i} \left[\prod_{(c \lor \tau_{m-i}) < t_{j} \le \tau_{m+1}} \lambda(t_{j} - \tau_{m-i}) \right] e^{-(\Lambda((c \lor \tau_{m+1}) - \tau_{m-i}) - \Lambda((c \lor \tau_{m-i}) - \tau_{m-i}))} \frac{L_{c \lor \tau_{m-i}}(\theta) L_{\tau_{m+1},T}(\theta)}{L_{T}(\theta)}$$

M step: $\lambda(t) = \alpha \beta t^{\beta-1}$

•
$$p_{k+1} = \left[\sum_{m=1}^{m_T} \pi_{m,0}^{\theta_k}\right] / m_T$$

• $\beta_{k+1} = \arg \max_{\beta} \left[n_T \left(\log(n_T \beta) - \log(S_k(\beta)) \right) + (\beta - 1) \right] \left[\sum_{\tau \in \{\tau_{m_c+1}, \dots, \tau_{m_T-\tau}, T\}} \sum_{i=0}^{m_{\tau^-}} \pi_{m_{\tau^-}, m_{\tau^-}-i}^{\theta_k} \sum_{(c \lor \tau_{m_{\tau^-}}) < t_j \le \tau} \log(t_j - \tau_i) \right] \right]$
with $S_k(\beta) = \sum_{\tau \in \{\tau_{m_c+1}, \dots, \tau_{m_T-\tau}, T\}} \sum_{i=0}^{m_{\tau^-}} \pi_{m_{\tau^-}, m_{\tau^-}-i}^{\theta_k} ((\tau - \tau_i)^\beta - ((c \lor \tau_{m_{\tau^-}}) - \tau_i)^\beta)$

• $\alpha_{k+1} = n_T / S_k(\beta_{k+1})$

VI. Reliability indicators

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• Failure intensity: $\lambda_t = \lim_{\Delta t \to 0} \frac{1}{\Delta t} P(N_{t+\Delta t} - N_{t-} | \underline{T}_{N_{t-}}, N_{t-})$

$$\lambda_t = \frac{\sum_{m=0}^{m_t-} D_{C_{K_{t-}}}^m e^{-(\Lambda(t-\tau_m) - \Lambda((c \vee \tau_m) - \tau_m)} L_{c \vee \tau_m}(\theta)}{\sum_{m=0}^{m_t-} D_{C_{K_{t-}}}^m \lambda(t-\tau_m) e^{-(\Lambda(t-\tau_m) - \Lambda((c \vee \tau_m) - \tau_m)} L_{c \vee \tau_m}(\theta)}$$

• Cumulative failure intensity: $\Lambda_t = \int_0^t \lambda_s \, ds$

$$\Lambda_t = -\left[\sum_{k=1}^{K_t} \log\left(\frac{L_{C_k^-}(\theta)}{L_{C_{k-1}^+}(\theta)}\right)\right] - \log\left(\frac{L_{t^-}(\theta)}{L_{C_{K_t}^+}(\theta)}\right)$$

- Reliability: $P(T_{N_T+1} > s \mid \underline{T}_{N_T}, N_T) = exp(-(\Lambda_s \Lambda_T))$
- Expected cumulative number of failures: $E[N_s | \underline{T}_{N_T}, N_T] =$

$$N_T + \left[\sum_{i=m_T+1}^{m_s} (\Lambda(s-\tau_i) + E[N_{\tau_i^-} | \underline{T}_{N_T}, N_T] - N_T)p(1-p)^{m_s-i}\right] \\ + \left[\sum_{i=0}^{m_T} (\Lambda(s-\tau_i) - \Lambda(T-\tau_i))(1-p)^{m_s-m_T} \pi_{m_T,m_T-i}^{\theta}\right]$$

VII. Application

The BP PM-ABAO CM model is implemented in **MARS** (Maintenance Assessment of Repairable Systems): a free software developped by LJK (Grenoble university) and EDF (French electricity utility).

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• MLE combined with moment estimation:

$$\tilde{\alpha} = 1.84 \times 10^{-6}, \quad \tilde{\beta} = 1.95, \quad \tilde{p} = 0.602, \quad log(\mathcal{L}_T(\tilde{\alpha}, \tilde{\beta}, \tilde{p})) = -150.900$$

• MLE computed with EM algorithm or direct maximization:

$$\hat{\alpha} = 1.87 \times 10^{-6}, \quad \hat{\beta} = 1.94, \quad \hat{p} = 0.614, \quad log(\mathcal{L}_T(\hat{\alpha}, \hat{\beta}, \hat{p})) = -150.902$$



Wearing out state at time $c \Rightarrow$ failures surge

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Quality of estimation: $\theta = (1.9e - 6, 1.9, 0.61)$

NEB: Normalized Empirical Bias NESD: Norm. Emp. Standard Deviation $\} \Rightarrow \begin{array}{l} \text{estimated over} \\ 60 \ 000 \ \text{simulations} \end{array}$ θ^* : MLE estimator when the B_i are known

Initial intensity:
$$\lambda(t) = \alpha \beta t^{\beta-1}$$
 or $\lambda(t) = \beta/\eta (t/\eta)^{\beta-1}$

EM is more robust than direct likelihood maximization

	NEB				NESD			
	α	η	eta	p	α	η	eta	p
$ ilde{ heta}$	590%	36%	8.2%	-9.4%	3300%	130%	35%	50%
$\hat{ heta}$	580%	37%	9.2%	-12%	3300%	130%	36%	51%
θ^*	18000%	-24%	-12%	0%	5000%	35%	22%	25%

Quality of individual PM efficiency estimation:

	Empirica	al mean	Empirical standard deviation		
$m^{ ext{th}} ext{ PM}$	$\pi_m^{ ilde{ heta}} - B_m$	$\tilde{p} - B_m$	$\pi_m^{ ilde{ heta}} - B_m$	$\tilde{p} - B_m$	
1	-7.8 e-2	-5.4 e-2	5.8 e-1	5.8 e-1	
2	-9.3 e-2	-5.8 e-2	5.9 e-1	5.8 e-1	
3	-9.4 e-2	-5.8 e-2	5.9 e-1	5.7 e-1	
4	-6.8 e-2	-5.8 e-2	6.0 e-1	5.6 e-1	
5	-4.3 e-2	-5.6 e-2	5.3 e-1	5.2 e-1	
6	-3.7 e-1	-6.1 e-2	6.4 e-1	5.8 e-1	
7	-1.3 e-2	-5.8 e-2	2.8 e-1	4.6 e-1	
8	7.2 e-3	-5.8 e-2	3.9 e-1	5.1 e-1	
9	9.4 e-4	-5.4 e-2	4.0 e-1	5.2 e-1	
10	-8.5 e-3	-5.5 e-2	4.7 e-1	5.5 e-1	
Mean	-7.5 e-2	-5.7 e-2	5.1 e-1	5.4 e-1	

 \Rightarrow Individual PM efficiency estimation is relevent after c

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Cumulative failure intensity and number of failures for $\theta = \hat{\theta}$











VIII. Prospects

- Maintenance times optimization
- Consider a more general distribution over $] \infty, 1]$ for the B_i .
- Develop confidence intervals and tests for the BP model

• ...

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