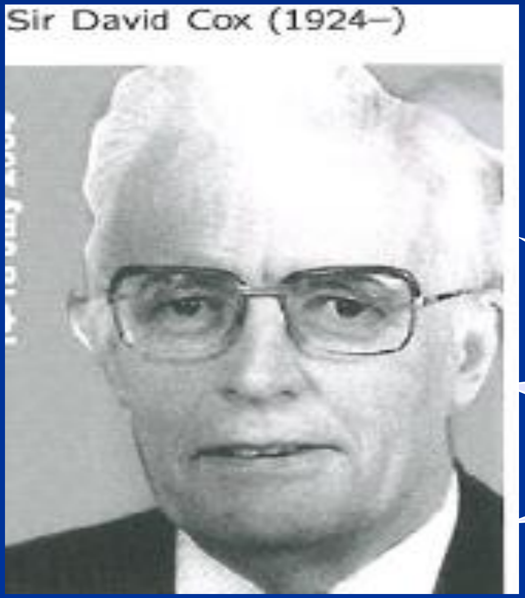
COMPLEMENT (?)

☛ Introduction à la Modélisation avec COVARIABLES

- ☛ VIBRATIONS,
- ☛ TEMPERATURES,
- ☛ CHARGES…



MODELES DE COX

SYSTEMES NON REPARABLES ☞ P.H.M.

SYSTEMES REPARABLES ☞ P.I.M, G.P.I.M, GRP_P.I.

PREAMBULE

- **SIMPLE EXEMPLE**

TRAITE AVEC

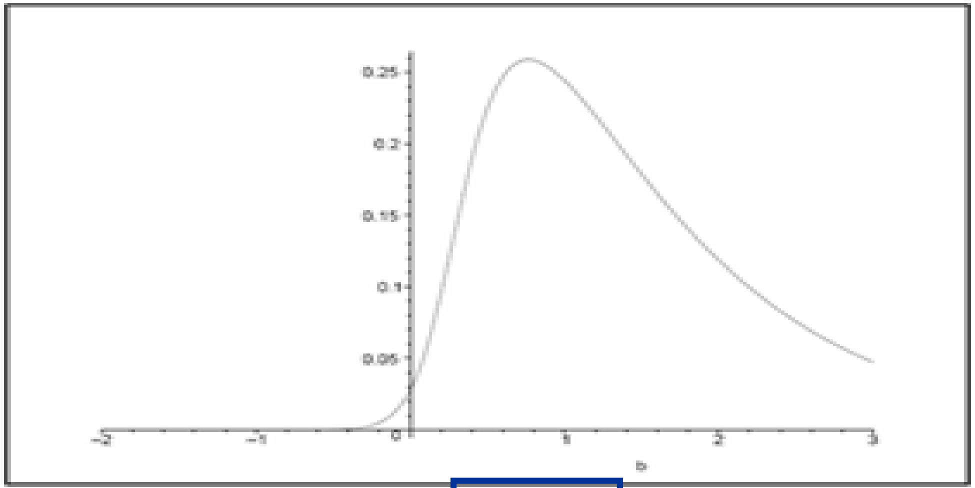
DIVERS OUTILS

SLIDE « cours BO. LINDQVIST »

SIMPLE EXAMPLE COX-REGRESSION

j	Y_j	x_j	δ_j
1	5	12	0
2	10	10	1
3	40	3	0
4	80	5	0
5	120	3	1
6	400	4	1
7	600	1	0

Cox' partial likelihood $L(\beta)$ in the example:



Maximum likelihood estimate: $\boxed{\hat{\beta} = 0.765.}$

Model:

- $z(t|x) = z_0(t)\exp\{\beta x\}$

Partial likelihood:

$$L(\beta) = \frac{e^{10\beta}}{e^{10\beta} + e^{3\beta} + e^{5\beta} + e^{3\beta} + e^{4\beta} + e^{\beta}} \cdot \frac{e^{3\beta}}{e^{3\beta} + e^{4\beta} + e^{\beta}} \cdot \frac{e^{4\beta}}{e^{4\beta} + e^{\beta}}$$

Weibull regression (Cox-example)

Minitab

```
Estimation Method: Maximum Likelihood

Distribution:   Weibull

Relationship with accelerating variable(s):    Linear


Regression Table

                           Standard                          95,0% Normal CI
Predictor      Coef        Error       Z       P          Lower        Upper
Intercept    7,58636    0,548229   13,84   0,000        6,51185      8,66087
x          -0,468235   0,0842830   -5,56   0,000      -0,633427    -0,303044
Shape        2,05563    0,872169                       0,894943      4,72167

Log-Likelihood = -17,450
```

Worksheet 1 ***

↓	C1	C2	C3
	Y	x	d
1	5	12	0
2	10	10	1
3	40	3	0
4	80	5	0
5	120	3	1
6	400	4	1
7	600	1	0

- *Cox proportional hazards models (Cox, 1972)*

SIMPLE EXAMPLE

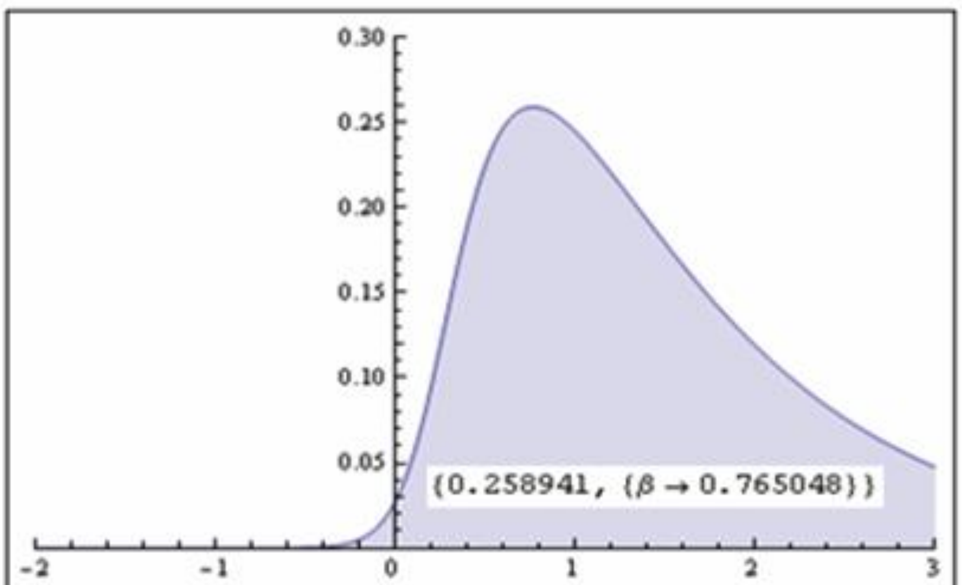
Code sous MATHEMATICA 7.0

```
xj = {12, 10, 3, 5, 3, 4, 1};
δj = {0, 1, 0, 0, 1, 1, 0};
z = {{10, 3, 5, 3, 4, 1}, {3, 4, 1}, {4, 1}};
LKV = (∏_{k=1}^{7} Exp[β * xj[[k]]]^δj[[k]]) /
   (Sum[Exp[β * z[[1, k]]], {k, 1, 6}] * Sum[Exp[β * z[[2, k]]], {k, 1, 3}] * Sum[Exp[β * z[[3, k]]], {k, 1, 2}]);
Print["LK_V(β) = ", LKV];

sol = NMaximize[LKV, {β, 0, 1}, MaxIterations → 1000, Method → {"NelderMead"}]
Framed@Plot[LKV, {β, -2, 3}, PlotRange → {{-2, 3}, {0, 0.3}}, Filling → Automatic]
```

Cox' partial likelihood

$$\text{LK\_V}(\beta) = \frac{e^{17\beta}}{\left(e^{\beta}+e^{4\beta}\right)\left(e^{\beta}+e^{3\beta}+e^{4\beta}\right)\left(e^{\beta}+2e^{3\beta}+e^{4\beta}+e^{5\beta}+e^{10\beta}\right)}$$



MATHEMATICA 9.0

```
xj = {12, 10, 3, 5, 3, 4, 1};
Yj = {5, 10, 40, 80, 120, 400, 600};
δj = {1, 0, 1, 1, 0, 0, 1};
e = EventData[Yj, δj];
𝒮 = CoxModelFit[{xj, e}, x, x];
𝒮["ParameterTable"]
```

	Estimate	Standard Error	Relative Risk	Wald-χ^2	DF	P-Value
x	0.765048	0.605674	2.1491	1.59551	1	0.20654

◆ PROPORTIONAL HAZARD WEIBULL
SIMPLE EXAMPLE

Code MATHEMATICA 7.0

```
xc = {5, 40, 80, 600}; xc[[0]] = 0;
Z1c = {12, 3, 5, 1}; nc = 4;
xf = {10, 120, 400}; xf[[0]] = 0;
Z1f = {10, 3, 4}; nf = 3;
Print[" WEIBULL AVEC COVARIABLES AVEC CENSURES "];
```

$$LLC = \sum_{i=1}^{nf} \text{Log}[\beta C * xf[[i]]^\wedge(\beta C - 1) * \text{Exp}[\gamma 0C + \gamma 1C * Z1f[[i]]] * \text{Exp}[-xf[[i]]^\wedge \beta C * \text{Exp}[\gamma 0C + \gamma 1C * Z1f[[i]]]]] - \sum_{i=1}^{nc} xc[[i]]^\wedge \beta C * \text{Exp}[\gamma 0C + \gamma 1C * Z1c[[i]]];$$

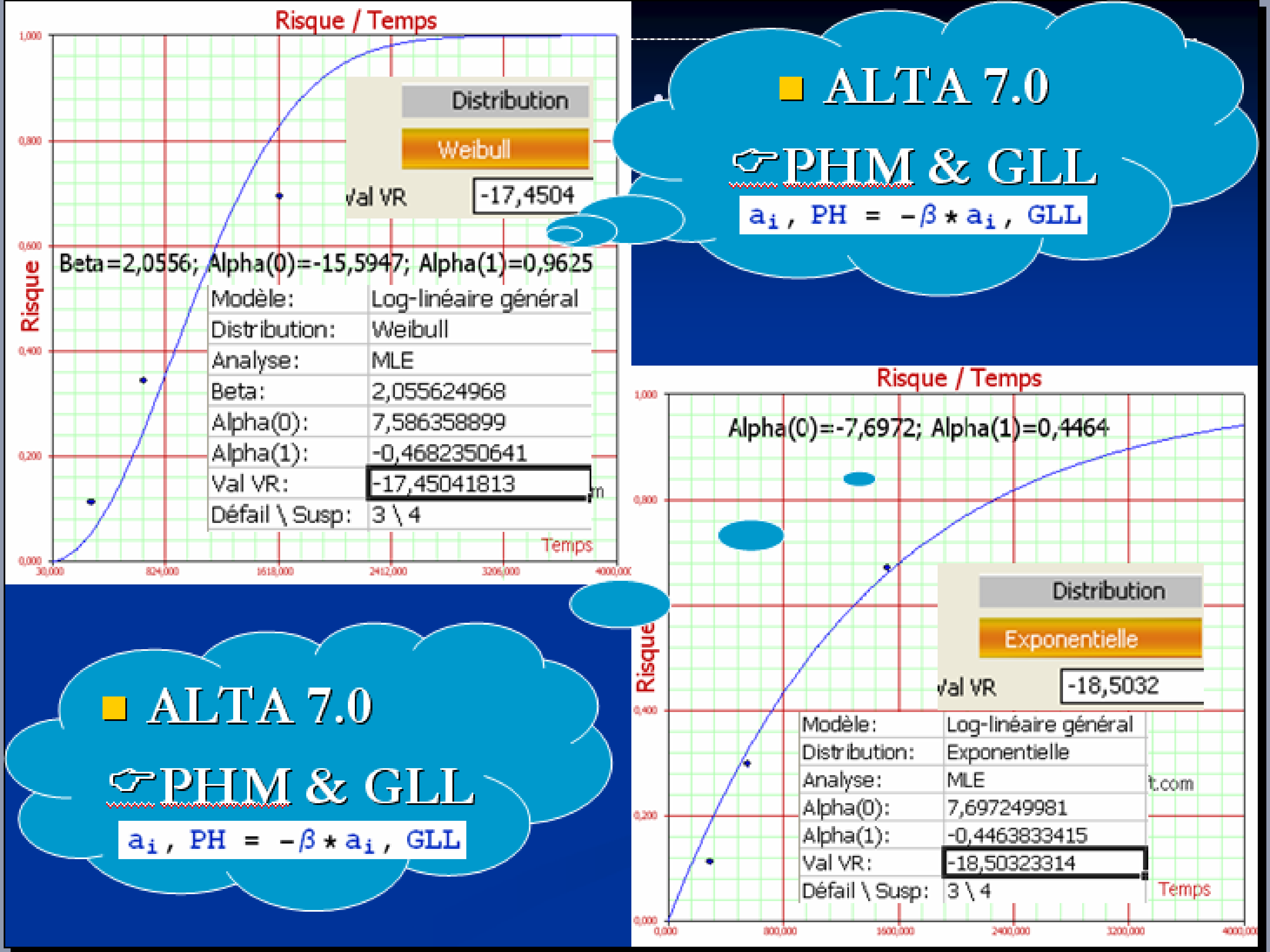
```
NMaximize[{LLC, 1 < βC < 3 && -20 < γ0C < 0 && 0 < γ1C < 1}, {{βC, 1, 3}, {γ0C, -20, 0}, {γ1C, 0, 1}},
 MaxIterations → 1000]
```

WEIBULL AVEC COVARIABLES AVEC CENSURES

$\boxed{\{-17.4504,}$ $\{\beta C \to 2.05563, \gamma 0C \to -15.5948, \gamma 1C \to 0.96252\}\}$

ALTA 7.0 de ReliaSoft

Modèle:	Hasards proportionnels
Distribution:	Weibull
Analyse:	MLE
Beta:	2,055624968
Alpha(0):	-15,59470877
Alpha(1):	0,9625156885
Val VR:	-17,45041813
Défail \ Susp:	3 \ 4



Risque / Temps
Distribution
Weibull
Val VR -17,4504
ALTA 7.0
☞PHM & GLL
a_i, PH = $-\beta * a_i$, GLL
Beta=2,0556; Alpha(0)=-15,5947; Alpha(1)=0,9625
Risque
Modèle: Log-linéaire général
Distribution: Weibull
Analyse: MLE
Beta: 2,055624968
Alpha(0): 7,586358899
Alpha(1): -0,4682350641
Val VR: -17,45041813
Défail \ Susp: 3 \ 4
Temps
Risque / Temps
Alpha(0)=-7,6972; Alpha(1)=0,4464
Distribution
Exponentielle
Val VR -18,5032
Risque
Modèle: Log-linéaire général
Distribution: Exponentielle
Analyse: MLE
Alpha(0): 7,697249981
Alpha(1): -0,4463833415
Val VR: -18,50323314
Défail \ Susp: 3 \ 4
Temps
ALTA 7.0
☞PHM & GLL
a_i, PH = $-\beta * a_i$, GLL

EXEMPLE « PHM » sur la Validité de l'Intensité de base & du Modèle Candidat

- « Regression with Life Data » de Minitab & « PHM » de ALTA 7.0 de ReliaSoft correspond à « LIFEREG » de SAS.
- « CoxModelFit » de Mathematica 9.0 (SYSTAT, SPSS, Origin…) correspond à « PHREG » de SAS.

☞ **Distinguo entre les deux estimations ?**

☞ **Cohérence des estimations ?**

APPLICATIONS AND CASE STUDIES

Reliability and operating environment-based spare parts estimation approach

EX :

A case study in Kiruna Mine, Sweden

Behzad Ghodrati and Uday Kumar
Division of Operation and Maintenance Engineering, Luleå University of Technology, Luleå, Sweden

Table AI contains the data sets used for demonstrating the concept. The column with TTFs exhibits time to failure of a particular type of hydraulic jack.

#	TTFs	MCSK	OPSK	ENDUS	HOILQ	STEMP
1	**2536**	1	1	-1	1	-1
2	**1200**	-1	-1	-1	1	-1
3	**3060**	1	1	1	1	1
4	**3652**	1	1	1	1	1
5	**2564**	1	1	1	1	-1
6	**644**	-1	-1	-1	-1	-1
7	**1380**	-1	-1	-1	1	-1
8	**2776**	1	1	1	1	-1
9	**1004**	-1	-1	-1	1	-1
10	**916**	-1	-1	-1	-1	-1
11	**2964**	1	1	1	1	1
12	**272**	-1	-1	-1	-1	-1
13	**1196**	-1	-1	-1	1	-1
14	**3920**	1	1	1	1	1
15	**2312**	1	-1	1	1	-1
16	**3696**	1	1	1	1	1
17	**3108**	1	1	1	1	1
18	**1216**	-1	-1	-1	1	-1
19	**2368**	1	-1	1	1	1
20	**2640**	1	-1	1	1	-1

The following abbreviations are used for denoting the covariates:

- **MSCSK:** Maintenance and service crew skill
- **OPSK:** Loader's Operator Skill
- **HOILQ:** Hydraulic Oil Quality
- **ENDUS:** Environmental factors and dust (specially)
- **STEMP:** System temperature (Hydraulic system)

(Kalbfleisch and Prentice, 1980):

$$L(\alpha)=\prod_{i=1}^{k}L_i(\alpha)=\prod_{i=1}^{k}\frac{\exp(S_i\alpha)}{\left[\sum_{m\in F(t_i)}\exp(z_m\alpha)\right]^{d_i}}$$

Table I. By following the step down procedure we found that the effects of three covariates (ENDUS, OPSK and STEMP) were significant at the 10 percent p-value.

Table I.
Estimation of covariates (the estimates of α and standard error (S.E.) were obtained by maximizing the likelihood function)

Final Model Summary Logiciel SYSTAT

Parameter	Estimate	S.E.	t-ratio	p-value
OPSK	-1.201	0.450	-2.668	0.008
ENDUS	-1.425	0.530	-2.689	0.007
STEMP	-0.748	0.444	-1.684	0.092

Based on the results from trend test, the time to failure cannot be exponentially distributed and on the other hand it follows the power law process with shape parameter $\beta_0 = 3$ and scale parameter $\eta_0 = 4,500$ hour (manufacturer recommendation).
With this assumption the hazard rate is equal to:

$$\lambda(t)=\frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1}\exp\left(\sum_{j=1}^{n}\alpha_j z_j\right)=\frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1}$$
$$\times\exp(-1.201\mathrm{OPSK}-1.425\mathrm{ENDUS}-0.748\mathrm{STEMP}).$$

N.B : Avec la recommandation du manufacturier on obtient : N(43424) # 10 défaillances qui représentent la moitié du REX (20 défaillances) cf. Table AI ?

VERIFICATION SOUS MATHEMATICA 9.0 (idem avec ORIGIN 7.0)

```
ClearAll["Global`*"];
ξ = {{1, -1, -1}, {-1, -1, -1}, {1, 1, 1}, {1, 1, 1}, {1, 1, -1}, {-1, -1, -1}, {-1, -1, -1}, {1, 1, -1},
   {-1, -1, -1}, {-1, -1, -1}, {1, 1, 1}, {-1, -1, -1}, {-1, -1, -1}, {1, 1, 1}, {-1, 1, -1}, {1, 1, 1},
   {1, 1, 1}, {-1, -1, -1}, {-1, 1, 1}, {-1, 1, -1}};
e = {2536, 1200, 3060, 3652, 2564, 644, 1380, 2776, 1004, 916, 2964, 272, 1196, 3920, 2312, 3696, 3108,
   1216, 2368, 2640};
e1 = EventData[20];
ℑ = CoxModelFit[{ξ, e}, {x1, x2, x3}, {x1, x2, x3}];
ℑ["ParameterTable"]
ℑ["TestTable"]
ℑ["RelativeRiskConfidenceIntervals"]
ℑ["ParameterConfidenceIntervals"]
```

X1 → OPSK
X2 → ENDUS
X3 → STEMP

	Estimate	Standard Error	Relative Risk	Wald-χ^2	DF	P-Value
x1	-1.39302	0.585048	0.248324	5.66934	1	0.017264
x2	-1.49703	0.631713	0.223795	5.6159	1	0.0177982
x3	-0.736612	0.663021	0.478733	1.23431	1	0.266571

CONSTAT : On note une estimation légèrement différente de la covariable « OPSK ».

AVEC COVARIABLES

MINITAB

Estimation Method: Maximum Likelihood

Distribution: Weibull

Regression Table

Predictor	Coef	Standard Error	Z	P	95,0% Normal CI Lower	95,0% Normal CI Upper
Intercept	7,63220	0,0540951	141,09	0,000	7,52618	7,73823
OPSK	0,254064	0,0543209	4,68	0,000	0,147597	0,360531
ENDUS	0,306972	0,0640041	4,80	0,000	0,181526	0,432418
STEMP	-0,0170885	0,0748282	-0,23	0,819	-0,163749	0,129572
Shape	4,79975	0,908700			3,31182	6,95617

Log-Likelihood = -152,016

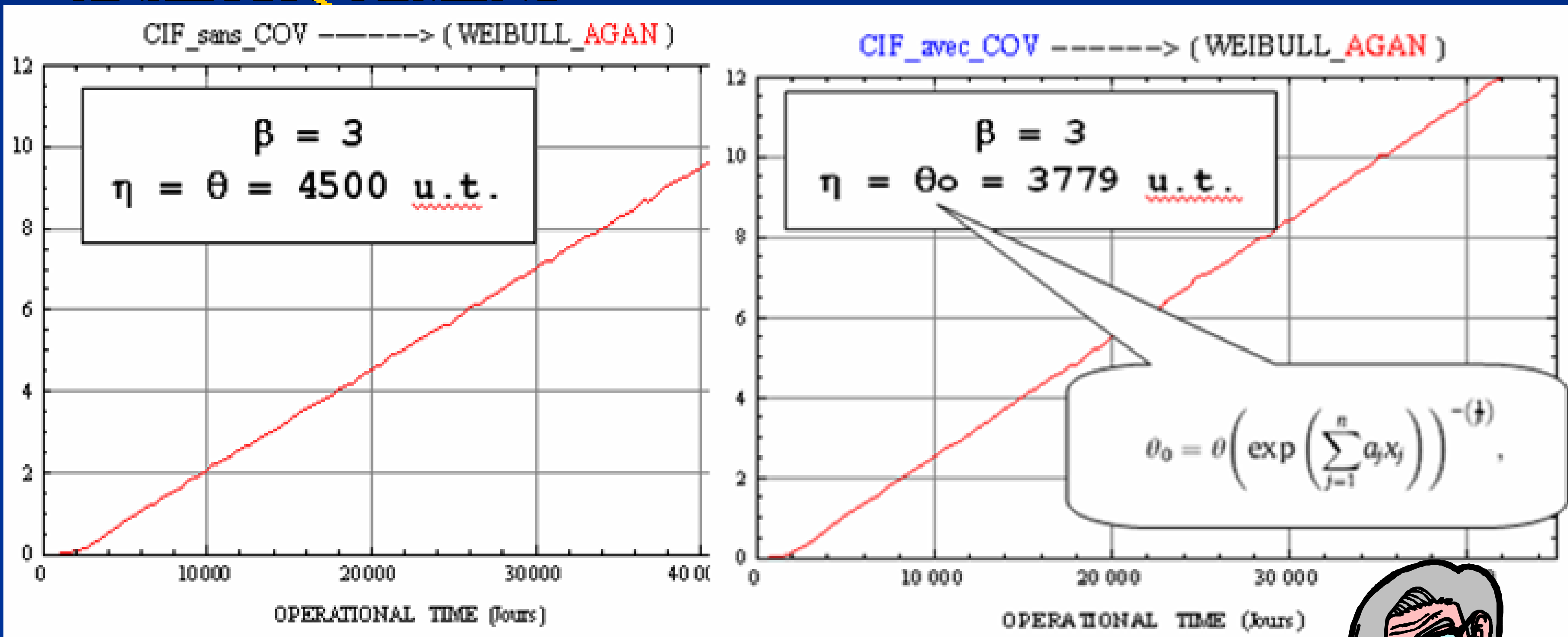
ALTA

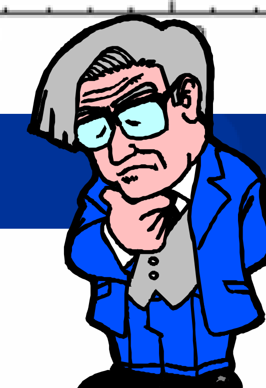
Modèle:	Hasards proportionnels
Distribution:	Weibull
Analyse:	MLE
Beta:	4,799746855
Alpha(0):	-36,63264587
Alpha(1):	-1,219444637
Alpha(2):	-1,473387798
Alpha(3):	0,0820202274
Val VR:	-152,0158038
Défail \ Susp:	20 \ 0

Modèle:	Hasards proportionnels
Distribution:	Exponentielle
Analyse:	MLE
Alpha(0):	-7,550129532
Alpha(1):	-0,2396894108
Alpha(2):	-0,3408182972
Alpha(3):	-0,01857190232
Val VR:	-171,093417
Défail \ Susp:	20 \ 0

ESTIMATIONS sans/avec COVARIABLES

- **ANALYTIQUEMENT**





- **ASYMPTOTIQUEMENT**

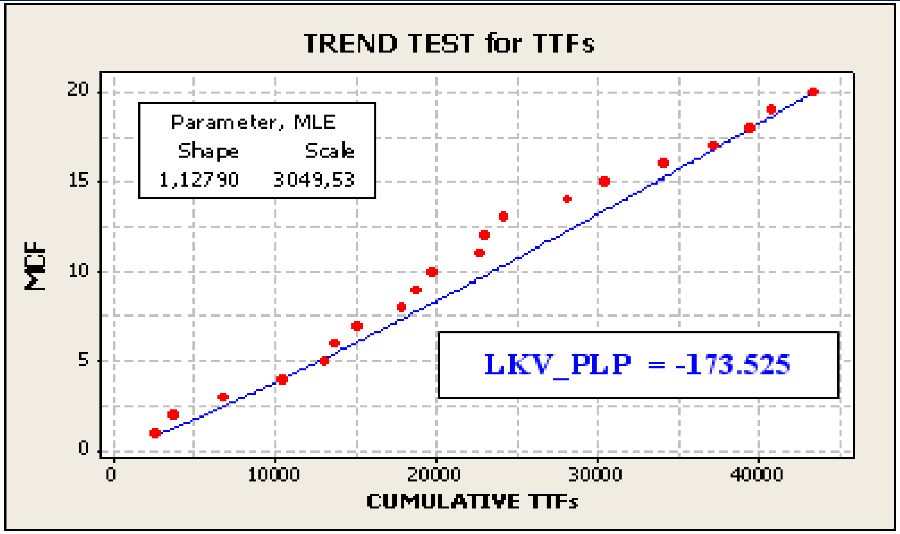
NF(T) = T/MUT + (var(T)-MUT²)/(2*MUT²)

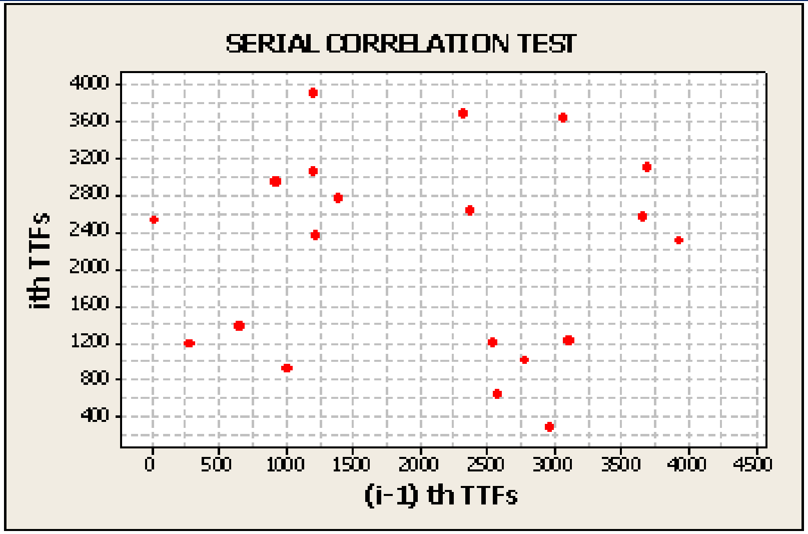
= T/MUT + 0.5*(1/β²-1)

Sans COV--> NF(43424)=(43424/4020)+(0.5*(0.364^2-1))= 10.3682 ?

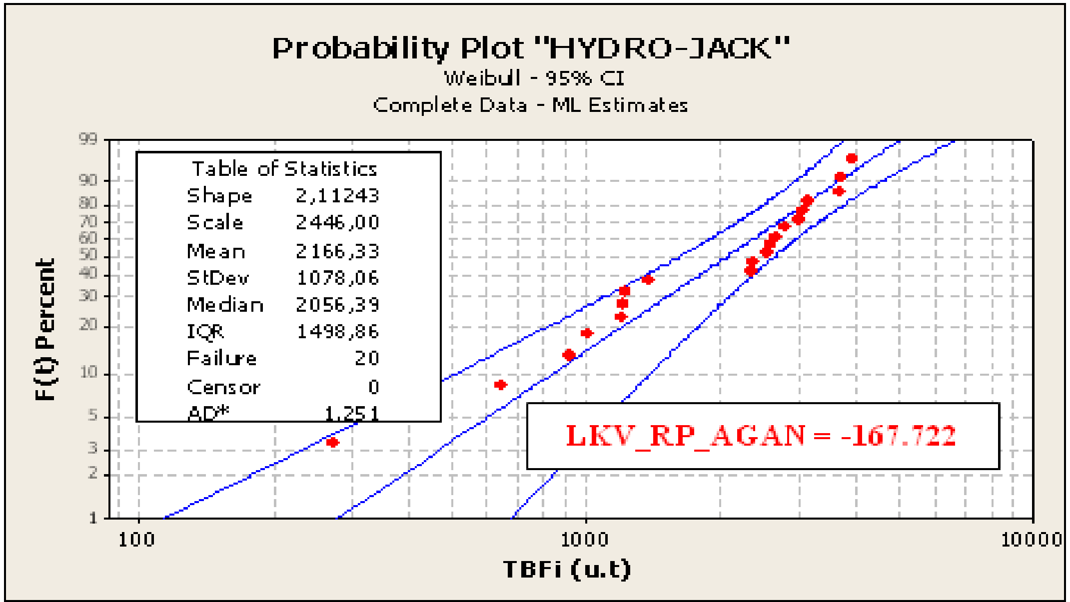
Avec COV--> NF(43424)=(43424/3374)+(0.5*(0.364^2-1))= 12.4364 ?

RE-ANALYSE (Protocole d'ASHER)





I.I.D. → RP_AGAN



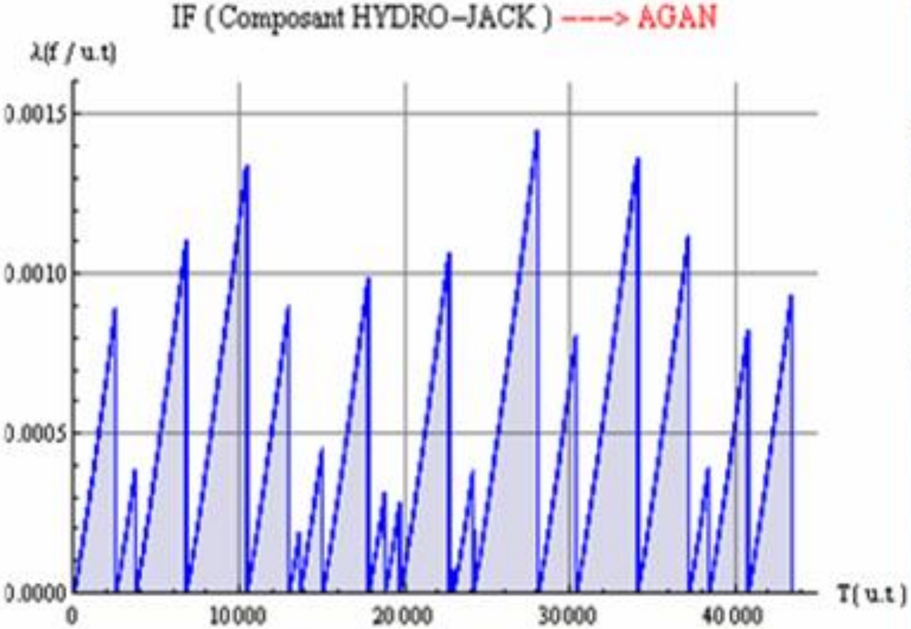
SANS COVARIABLES

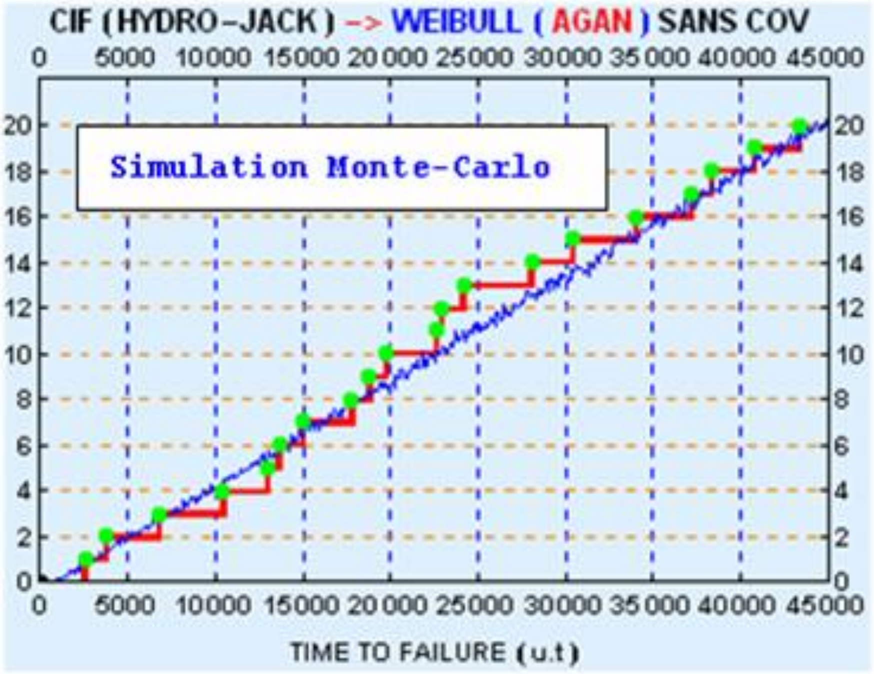
REX = {2536,3736,6796,10448,13012,13656,15036,17812,18816,19732,
22696,22968,24164,28084,30396,34092,37200,38416,40784,43424}u.t.

NF=20;

MODELE PARAMETRIQUE → WEIBULL

β = 2.11243; λ = 6.95*10^-8 déf./u.t.; η = 2446 u.t.;



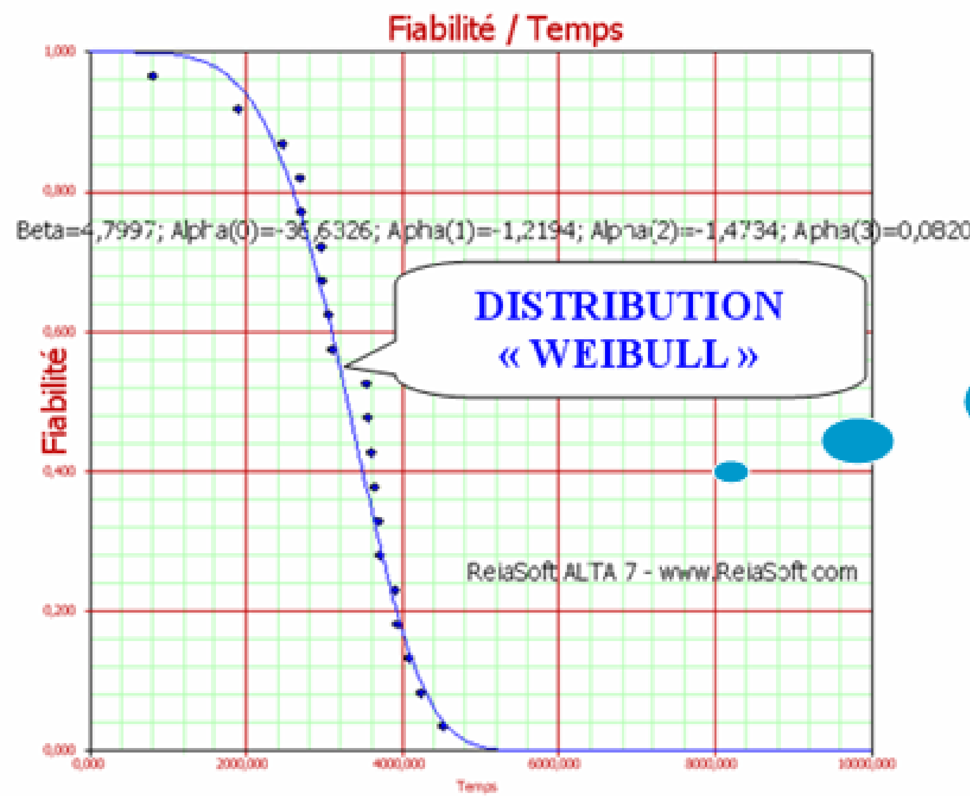


Par Intégration de « IF »

Nbre de défaillances de 0 à 43424 u.t = 20 déf.

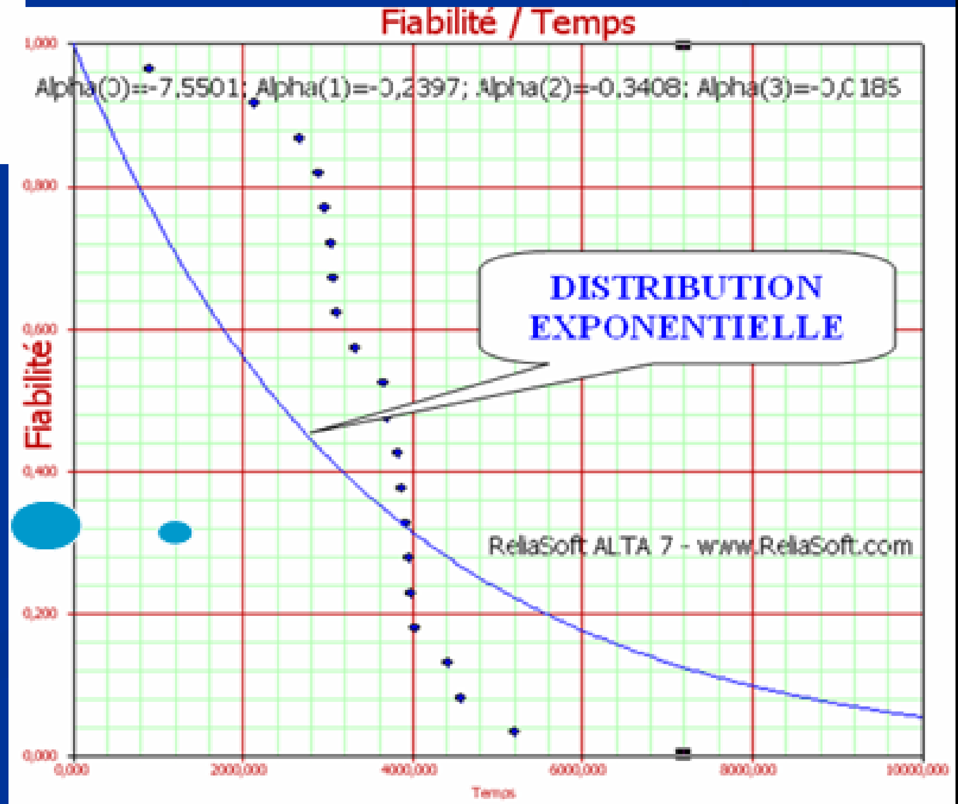
«Asymptotiquement»

Sans COV-->N(43424)=(43424/2116)+(0.51*(0.51^2-1))= 20.15 déf.


Fiabilité / Temps
Beta=4,7997; Alpha(0)=-36,6326; Alpha(1)=-1,2194; Alpha(2)=-1,4734; Alpha(3)=0,0820
DISTRIBUTION
« WEIBULL »
ReliaSoft ALTA 7 - www.ReliaSoft.com
Fiabilité
1,000
0,800
0,600
0,400
0,200
0,000
0,000
2000,000
4000,000
6000,000
8000,000
10000,000
Temps


ALTA 7.0
ReliaSoft


ALTA 7.0
ReliaSoft


Fiabilité / Temps
Alpha(0)=-7,5501; Alpha(1)=-0,2397; Alpha(2)=-0,3408; Alpha(3)=-0,0185
DISTRIBUTION
EXPONENTIELLE
ReliaSoft ALTA 7 - www.ReliaSoft.com
Fiabilité
1,000
0,800
0,600
0,400
0,200
0,000
0,000
2000,000
4000,000
6000,000
8000,000
10000,000
Temps

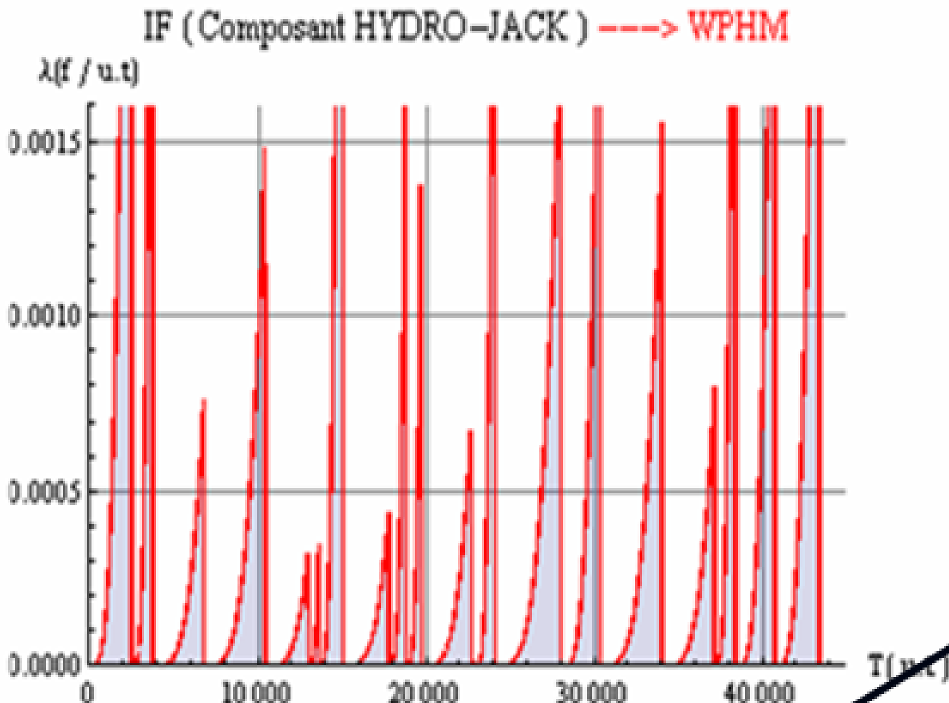
--> VERIFICATION du modèle Semi-Paramètrique «WPHM (ALTA)»:

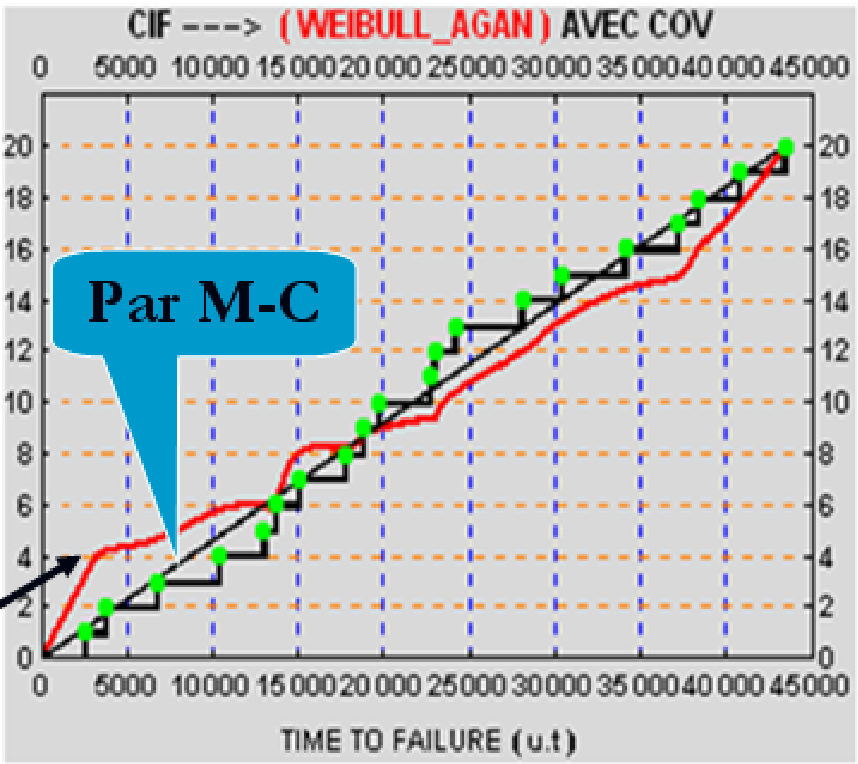
x1 = OPSK = -4.79975*0.254064 = -1.21944

x2 = ENDUS = -4.79975*0.306972 = -1.47338

x3 = STEMP = -4.79975*(-0.0170885)= 0.082020

η = Exp[7.63220] = 2063.5849 u.t.





Par Intégration de « IF »

COHERENCE AVEC REX

Nbre de défaillances de 0 à 43424 u.t = 20 déf.

EXEMPLE « PIM »

- Le papier de « FUQING » utilise le REX de « GHODRATI » de l'EX. (en divisant les données par 500).
- Nous soulignons le manque de cohérence de sa « Intensity Function » dont les paramètres ont été calculés par MCMC.
- Nous observons une limitation des Modèles mis en œuvre et une sélection desdits modèles seulement par « BIC ».

☛ N.B : cf. Extension avec le papier « Imperfect Repair Proportional Intensity Models for Maintained Systems »
by A. SYAMSUNDAR. IEEE 2011

International Journal of Performability Engineering Vol. 9, No. 2, March 2013, pp.163-174.

Proportional Intensity Model considering Imperfect Repair for Repairable Systems

YUAN FUOING*and UDAY KUMAR
Division of Operation and Maintenance
Luleå University of Technology, SE-971 87Lulea, SWEDEN

Numerical Example

In order to demonstrate the methodology proposed above, we introduce an example which has been discussed by Ghodrati and Kumar . The hydraulic brake pump is a critical part of the hydraulic loader. It is known that the following factors can influence its reliability: the operator skill (OPSK), maintenance crew skill (SCSK), hydraulic oil quality (HOILQ), hydraulic system temperature (STEMP), and environmental conditions (ENDUS). Our paper uses the data in Table AI from their paper

The best model is the Kijima II based PIM model with the covariates $\gamma_1\ \gamma_2\ \gamma_4$, corresponding to BIC=54. The detailed results for this model are tabulated in Table 2 where SD is standard deviation.

Table 2: Evaluated parameter values

Parameter	Mean	SD	Lower bound (0.025)	Upper bound (0.975)
Scale Parameter α	3.7878	0.2833	3.2936	4.41
Shape Parameter β	7.83	1.29	5.5	10.6
Imperfect Repair Factor q	0.099	0.0394	0.0260	0.1838
OPSK β_1	-2.584	0.6212	-3.81	-1.55
SCSK β_2	-0.978	0.3722	-1.71	-0.27
STEMP β_4	-1.51	0.4155	-2.3538	-0.6831

```
REX = {5.072, 7.472, 13.592, 20.896, 26.024, 27.312, 30.072, 35.624, 37.632, 39.464, 45.392,
  45.936, 48.328, 56.168, 60.792, 68.184, 74.4, 76.832, 81.568, 86.848}; t[[0]] = 0; NF = 20;
Z1 = {1, -1, 1, 1, 1, -1, -1, 1, -1, -1, 1, -1, -1, 1, -1, 1, 1, -1, -1, -1}; (* OPSK *)
Z2 = {1, -1, 1, 1, 1, -1, -1, 1, -1, -1, 1, -1, -1, 1, 1, 1, 1, -1, 1, 1}; (* SCSK *)
Z3 = {-1, -1, 1, 1, -1, -1, -1, -1, -1, -1, 1, -1, -1, 1, -1, 1, 1, -1, 1, -1}; (* STEMP *)
```

In the optimal PIM model, the mean for q is 0.099. In order to demonstrate the effectiveness of repair, we plot its intensity function against time with $q = 0.099$, as shown in Figure 2, where the intensity function is for the 14th to the 17th failure. We can see from this figure that repair is significantly effective as the system's intensity function has almost been restored to zero after each repair.

$$\lambda(x_i, z; q) = \frac{\beta}{\alpha}\left(\frac{V_{i-1} + x_i}{\alpha}\right)^{\beta-1} \exp(\gamma_1 z_{1i} + \gamma_2 z_{2i} + \gamma_3 z_{3i} + \ldots b)$$

KIJIMA_II

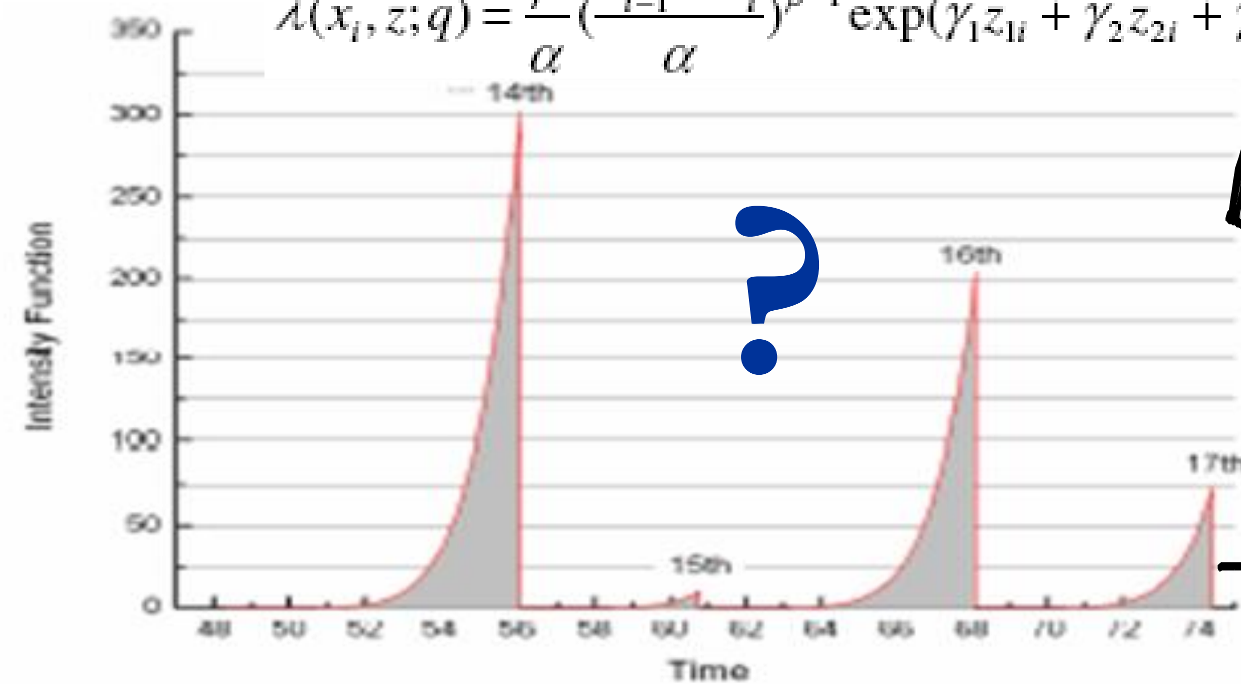


Figure 2: Effectiveness of Repair

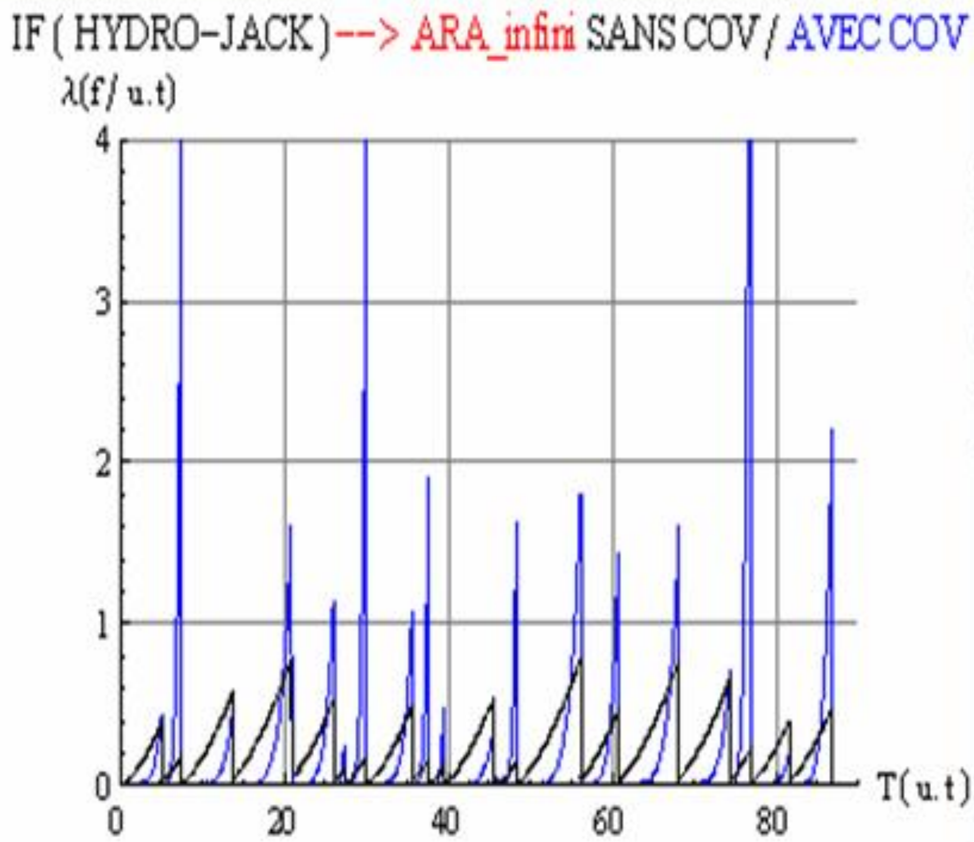
RE-ANALYSE KIJIMA_II (ARA∞) sans/avec COVARIABLES

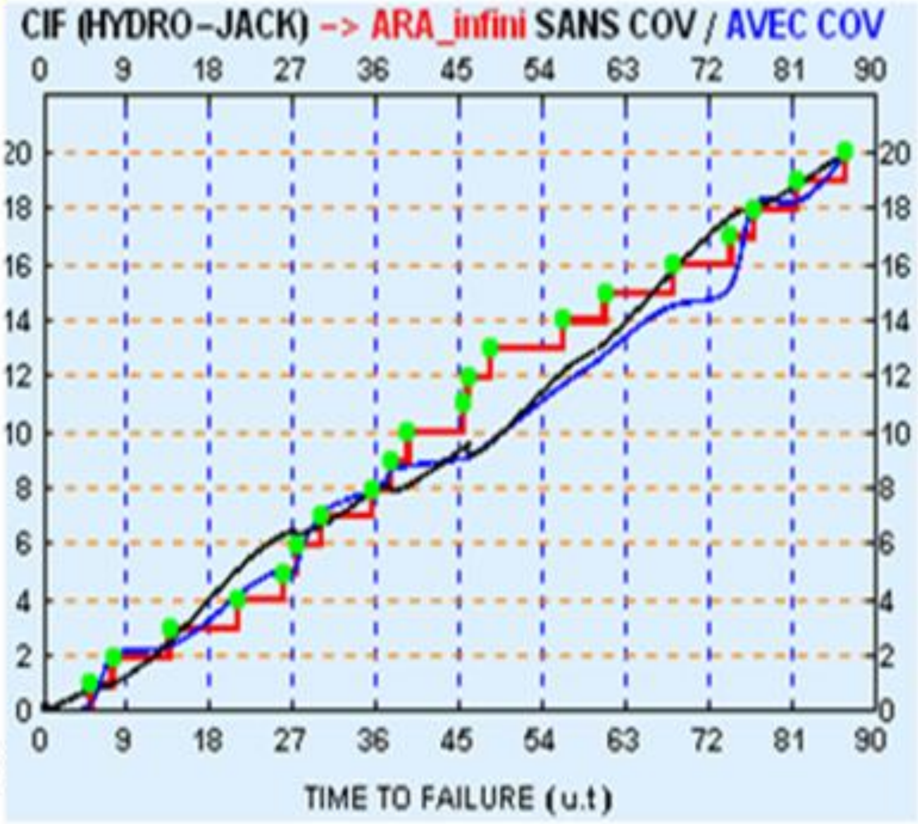
ARAinfini Sans COVARIABLES

$\beta = 2.48785$; $\alpha = 0.013749$; $\rho = 0.862166$;

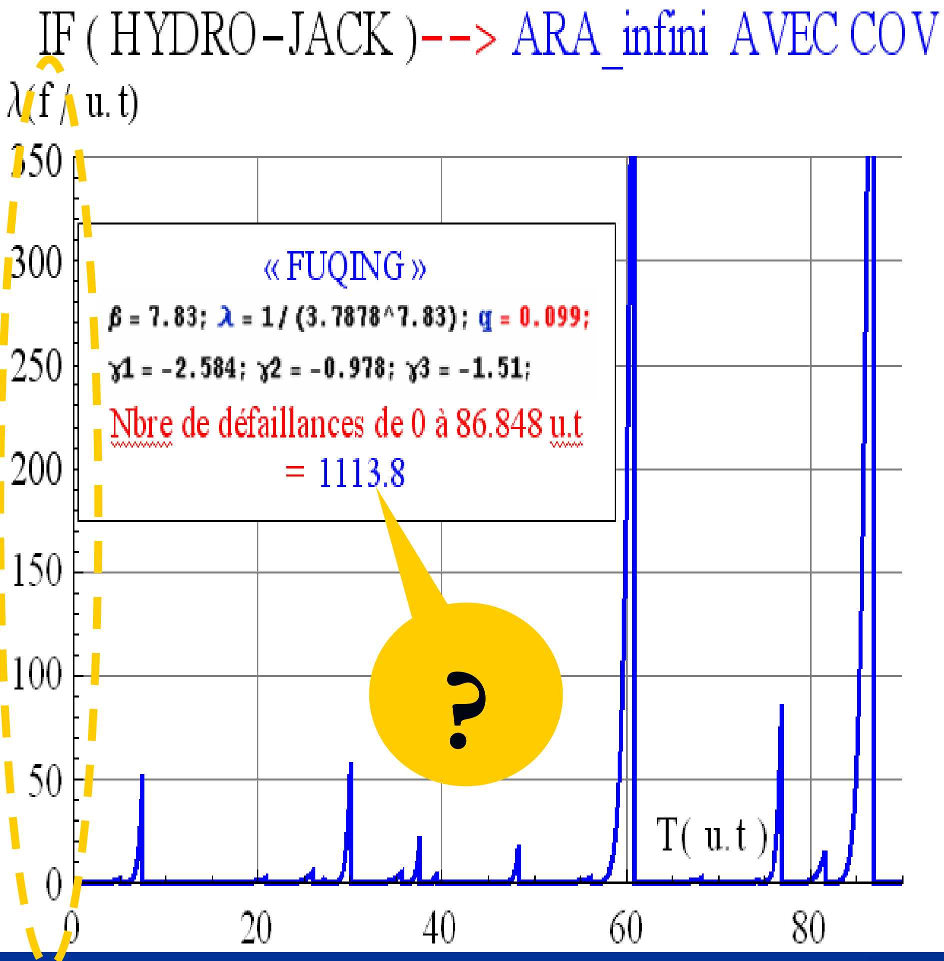
ARAinfini avec COVARIABLES

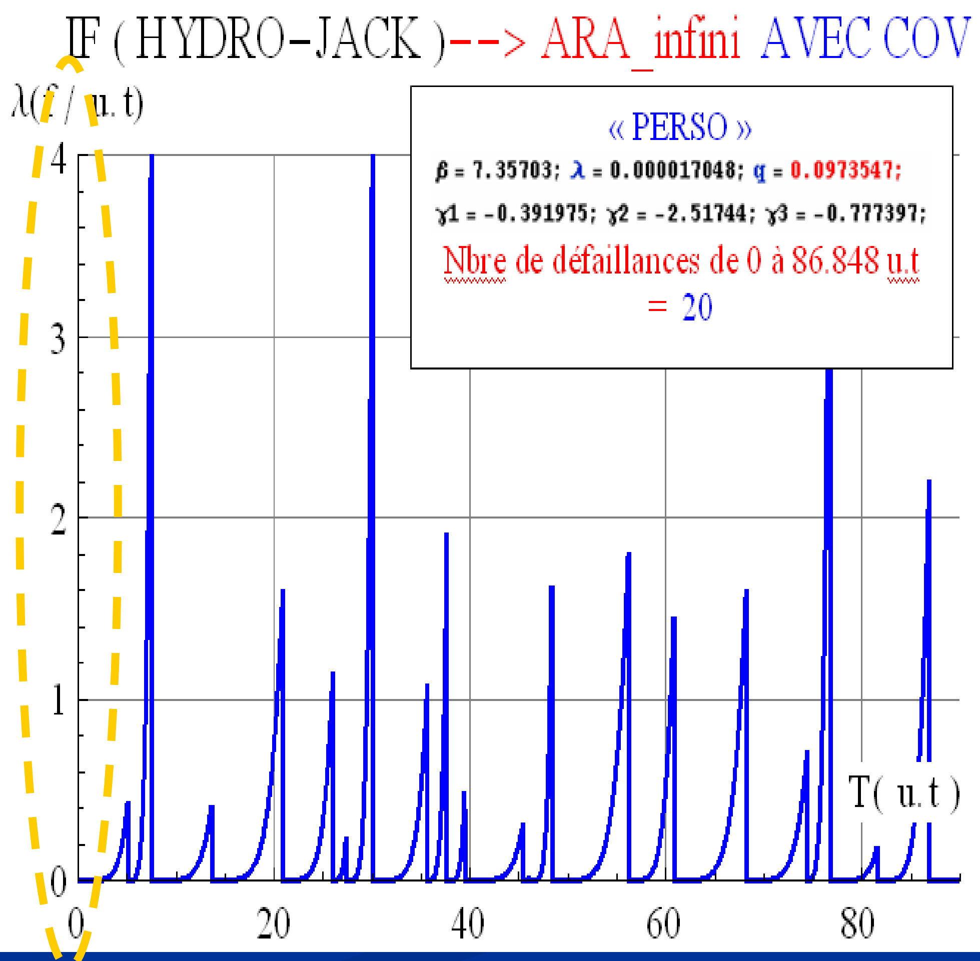
$\beta = 7.35703$; $\alpha = 0.000017048$; $\rho = 0.9026453$; $\gamma 1 = -0.391975$; $\gamma 2 = -2.51744$; $\gamma 3 = -0.777397$;





DISTINGUO FUQING/PERSO





MERCI DE VOTRE ATTENTION

Un modèle simple
est faux.
Un modèle complexe est inutilisable.

dixit « Paul VALERY »