Degradation data analysis for samples under unequal operating conditions: a case study on train wheels

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Summary

- Motivating situation
- Reliability data - main sources and types
- Degradation data to access reliability- characteristics, prerequisites and practical advantages
- Literature on degradation data analysis
- General Degradation Path Models (assumptions and parameter estimation)
- Train wheel degradation data revisited
- Conclusions

Motivating situation

- Data set: diameter measures of trains wheels
  - ✓ folow up time: 600,000 Km;
  - ✓ inspection times: each 50,000 Km (0, 50,000; 100,000; 150,000 ... 600,000);
  - ✓ 14 locomotives ($8 \times 14 = 112$ wheels);

Motivating situation(cont.)

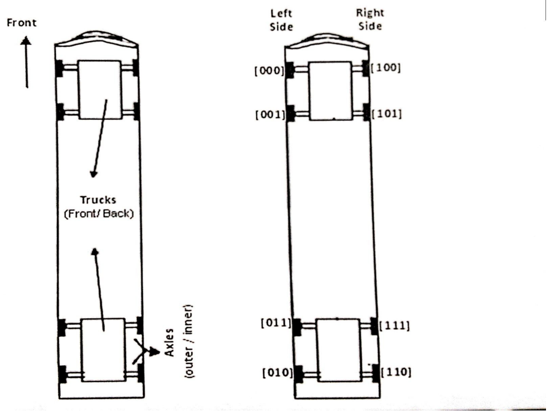

Figure : The location of the wheels: side, axles and trucks within a car and their corresponding labels.

Motivating situation (cont.)

- Degradation measurementes$\Rightarrow$ 966 mm-[observed diameter at time (Km) t];
- threshold level (defines the *failure* (D_f) $\Rightarrow$ a given wheel is replaced when its diameter reaches 889 mm $\Rightarrow$ D_f=77 mm;

Motivating situation (cont)

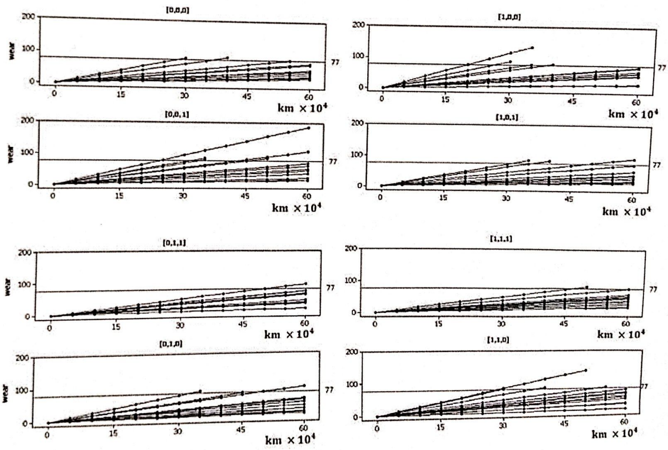

Figure : Wheels degradation profiles by working positions.

Motivating Situation (cont.)

- Questions:

  ✓ do the different working positions have significant effect on the wheels wear?

  ✓ if that is the case, what is the time-to-failure distribution of wheels on different working positions?

  ✓ in addition, it is important to get estimates of key reliability summary figures, such as:

    - the MTTF (mean time to failure, or more specifically, mean distance to failure)
    - quantiles of the time-to-failure distribution (e.g. 0.01, 0.10 and the median, 0.50)

Reliability Data - Main sources and Types

- Main sources:
  - ✓ tests (life tests, accelerated life testes, degradation tests, accelerated degradation testes, etc.)
  - ✓ field data (includes warranty returns data)
  - ✓ handbooks (manuais)

Reliability Data - Main sources and Types (cont.)

- Types of reliability data:
  - ✓ failure times+ censored failure times
  - ✓ degradation data : a product characteristic whose degradation over time can be closely related to failure (examples: loss of tread on rubber tires and degradation of the active ingredient of a drug because of chemical reactions with oxygen and water or microbial)
  - ✓ records of recurrent events: recurrent failure times + times of maitenance actions (*maintenance data*)

Degradation data to access reliability- characteristics, prerequisites and practical advantages

- Characteristics and prerequisites:

  ✓ the existance of a product characteristic whose degradation over time can be closely related to failure and can be accurately measured.

  EXAMPLES : crack size growth over time; the amount of wear over time (Km) on train wheels; luminosity of fluorescent lights or luminous flux, etc.)

Degradation data to access reliability- characteristics, prerequisites and practical advantages (cont.)

✓ prespecification of a "threshold" level of degradation $\rightarrow$ failure occurs when the amount of degradation for a test unit exceeds this level. Examples: failure is defined to occur when:

- the crack reaches a length of 2cm
- the diameter measure of the wheels reaches a predefined value of 889 mm
- a lamp's luminous flux falls below 60% of its initial luminous flux, after 100 hours of use.

Degradation data to access reliability- characteristics, prerequisites and practical advantages (cont.)

- Practical advantages:
  - Most failures arise from degradation mechanisms at work for which there are characteristics that degrade (or grow) over time (e.g., amount of material displaced by electro migration)
  - Degradation data can be analyzed earlier, before a failure actually occurs
  - Degradation data may yield more accurate life estimates than the accelerated life tests with few or no failures
  - Degradation data can provide better information of degradation processes, which helps one to find the appropriate mechanistic model for degradation

Literature on degradation data analysis

- Stochastic Process
  - Birnbaum and Saunders (1969); Battacharyya and Fries (1982); Doksum (1991); Whitmore (1995); Doksum and Normand (1995); Whitmore and Schenkelberg (1997); Padgett (2004): **Wiener process**

Literature on degradation data analysis (cont.)

- Bagdonavicius and Nikulin (2000); Lawless and Crowder (2004):**gamma process**
- Pan e Balakrishanan (2011): two degradation characteristics - modeled by Bivariate Birnbaum-Saunders;
- Ng (2008): monotonic degradation profiles with one random change point.

Literature on degradation data anlysis (cont.)

- General Degradation Path Models
  - methods: Nelson (1990) (chapter 11); Carey e Koenig (1991); Lu and Meeker (1993); Lu et al. (1997); Su et al. (1999); Almeida (2011)
  - applications: Tseng, Hamada and Chiao (1995); Yacout, Salvatores and Orechwa (1996); Lu et al. (1997); Su et al. (1999); Wu e Shao (1999); Wu and Tsai (2000); Crk (2000); Oliveira and Colosimo (2004); Freitas et al. (2009); Peng e Tseng (2009); Ferreira, Freitas and Colosimo (2011)

Literature on degradation data analysis (cont.)

- basic reference: Meeker and Escobar (1998, cap.13 e 21);
- Bayesian approach: Hamada(2008) - book ; Hamada (2005) - applied bayesian inference to deal with the random effects of the general degradation path models - laser data; Freitas et.al. (2010) - same approach as in Hamada (2005) applied to the data of train wheels (wheel at position [000]);

General Degradation Path Models (assumptions and parameter estimation)

- General form:

$$Y_{ij} = D_{ij} + \varepsilon_{ij} = D(t_{ij}; \alpha; \beta_i) + \varepsilon_{ij},$$

with $i = 1, 2, \ldots, n$ e $j = 1, 2, \ldots, m_i$,

  - Y_{ij} is a random variable representing the amount of degradation of the i^{th} unit at a prespecified time t_{ij} $(i = 1, 2, \ldots, n; j = 1, 2, \ldots, m_i)$;

General Degradation Path Models (assumptions and parameter estimation)

- $D(t_{ij}; \alpha; \beta_i)$ is the theoretical degradation path of unit i at time t_{ij} (linear or nonlinear form);

- ε_{ij} is the random error associated to the i^{th} unit at time t_{ij};

- $\alpha = (\alpha_1; \alpha_2; \ldots; \alpha_p)^t$ is a $p \times 1$ vector of fixed effects describing population characteristics (they are modeled as common across all units);

- $\beta_i = (\beta_{i1}, \beta_{i2}, \ldots, \beta_{ik})^t$ is a $k \times 1$ vector or the i^{th} unit random effects representing the individual unit's characteristics (variations in the manufacturing of the components, such as properties of the raw material and component dimensions).

General Degradation Path Models (assumptions and parameter estimation)

- Assumptions:
  - ε_{ij} are *i.i.d.* Normal $(0;\sigma_\varepsilon^2)$ (σ_ε^2 fix and unknown)
  - $\beta_i = (\beta_{i1}, \beta_{i2}, \ldots, \beta_{ik})^t$ $i = 1, \ \ldots, \ n$ are random vectors *i.i.d.* with a multivariate distribution $\Lambda(\beta|\theta)$ and density function $f(\beta|\theta)$, indexed by a parameter vector θ $(q \times 1)$ fixed and unknown (which needs to be estimated using the data)
  - β_i e ε_{ij} are independent.

General Degradation Path Models (assumptions and parameter estimation) (cont.)

MAIN CHARACTERISTIC OF THE DATA ANALYSIS

$\Downarrow$

TWO STAGES

- STAGE 1: model fitting to the degradation data (lontigudinal data) - parameter estimation
- STAGE 2 : evaluation of the time to failure distribution and other key reliability figures (MTTF, quantiles)

General Degradation Path Models (assumptions and parameter estimation) (cont.)

STAGE 1: Model fitting to the degradation data (lontigudinal data) - parameter estimation: Likelihood function

- Model parameters: α, θ and σ_ε^2 (all fixed and unknown)

$$f(y|\alpha,\ \theta,\ \sigma_\varepsilon^2) = \prod_{i=1}^{n} \left\{ \int_{\Xi_{\beta_i}} \left[\prod_{j=1}^{m_i} \frac{1}{\sigma_\varepsilon}\ \phi_{NOR}(z_{ij}) \right]\ f(\beta_i|\theta)\mathrm{d}\beta_i \right\},$$

where $z_{ij} = \frac{[y_{ij} - D(t_{ij},\ \alpha,\ \beta_i)]}{\sigma_\varepsilon}$ e ϕ_{NOR} is the density of a Standard Normal distribution function.

General Degradation Path Models (assumptions and parameter estimation) (cont.)

- REMARK: Usual assumption:
  - ✓ $\beta_i = (\beta_{i1}, \beta_{i2}, \ldots, \beta_{ik})^t \;\; (i = 1, \; \ldots, \; n \;)$ *i.i.d.* $N_k(\mu_\beta, \; \Sigma_\beta)$
  - ✓ model parameters: μ_β; Σ_β; α, σ_ε^2 (fixed and unknown)
  - ✓ softwares SAS, MATLAB, R (lme, nlme) etc. Model fitting assuming Normal distribution.

General Degradation Path Models (assumptions and parameter estimation) (cont.)

STAGE 2: Evaluation of the time to failure distribution F(t)

A specified model for $D(t; \alpha, \beta)$ and D_f defines a failure time distribution. In general, this distribution can be written as a function of the degradation model parameters.

General Degradation Path Models (assumptions and parameter estimation) (cont.)

- Suppose that a unit fails at time t if the degradation level reaches D_f at time t. Then:
  - degradation measurements *increasing* with time

$$F_T(t) = P(T \leq t) = P[D(t;\alpha;\beta) \geq D_f]$$

  - degradation measurements *decreasing* with time

$$F_T(t) = P(T \leq t) = P[D(t;\alpha;\beta) \leq D_f]$$

General Degradation Path Models (assumptions and parameter estimation) (cont.)

- Three procedures that might be used to evaluate $F_T(t)$:
  - ✓ Analytical solution
  - ✓ Direct integration
  - ✓ Monte Carlo simulation

General Degradation Path Models (assumptions and parameter estimation) (cont.)

- Evaluation of $F(t)$ by Monte Carlo simulation:
  - Using the parameter estimates $\hat{\alpha}$, $\hat{\theta}$ and $\hat{\sigma}_{\varepsilon}^2$ generate M degradation profiles $D(t)$ and compute the "pseudo failure times" t_j^* $(j = 1, \ldots, M)$ using $y = D_f$.
  - Then, for any desired values of t, use the proportion of paths crossing D_f by time t as an evaluation of $F(t)$. In other words:

$$\hat{F}_T(t) = \frac{\sum_{j=1}^{M} I_{(t_{j^*} \leq t)}}{M} \qquad t > 0$$

  - Confidence intervals can be obtained using the bias-corrected bootstrap method.

Train wheel degradation data revisited

1) Empirical search with an approximate degradation analysis performed with the *PSEUDO* FAILURE TIMES (Meeker and Escobar, 1998

- Approximate analysis
  - ✓ least squares fit (straight line) to each wheel profile:

$$y_{ij} = D_i\left(t_{ij};\beta_{0i};\beta_{1i}\right) + \epsilon_{ij} = \beta_{0i} + \beta_{1i} t_{ij} + \varepsilon_{ij} \qquad (j = 1, \ldots, 13)$$

Train wheel degradation data revisited (cont.)

- ✓ Calculation of the *pseudo failure times* (Meeker and Escobar, 1998) for each wheel profile

$$\hat{t}_i = \frac{D_f - \hat{\beta}_{0i}}{\hat{\beta}_{1i}}$$

- ✓ lifetime data analysis using the pseudo failure times (search of distributions - Weibull, etc.)
- ✓ distribution fitting to the pseudo failure times

Train wheel degradation data revisited (cont.)

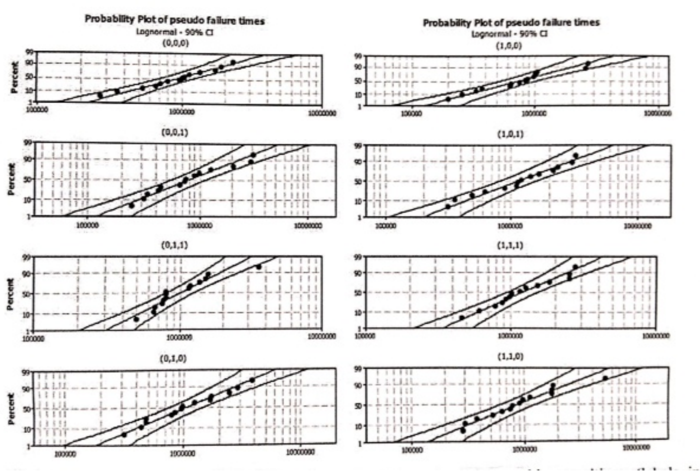

Figure : Lognormal probability plots based on pseudo failure times, by working positions

Train wheel degradation data revisited (cont.)

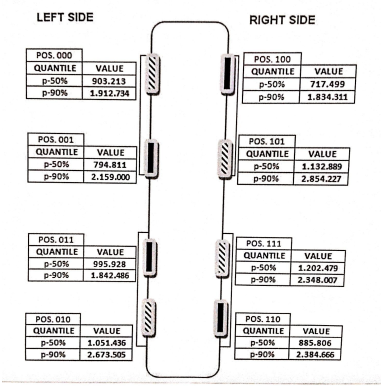

Figure : Point estimates of quantiles (0.50 and 0.90) of the failure time distributions based on pseudo-failure time data (by working positions). [higher values indicated by a hachured area] [axle ×side???]

Train wheel degradation data revisited (cont.)

- Preliminary conclusions based on the empirical search
  - The patterns of the degradation profiles (Figure 2) suggest a linear (straight line) functional form for the degradation path for all the working positions, with a positive degradation rate (slope).
  - The lognormal distribution is a good candidate for the distribution of time-to-failure distributions of the wheels.

Train wheel degradation data revisited (cont.)

- As a consequence of 1 and 2, the degradation rate (the slope) should also have a lognormal distribution
- There is an indication of interactions between the working condition factors (in particular, side and axle) that might be affecting the degradation rate and should be included in the model, possibly by writing the degradation rate as a function of these factors and interactions.

Train wheel degradation data revisited (cont.)

2) MODEL SPECIFICATION

Nonlinear Mixed Effects Model (NLME) for the i^{th} unit:

$$Y_{ij} = \alpha_0 + e^{\eta_i} \; t_j + \varepsilon_{ij} \quad (i = 1, \ldots, n; j = 1, \ldots, m_i)$$

$$\eta_i = \eta \left(X_{ij}^t \; , \; \alpha \; , \; \beta_i \right) = \beta_i + X_{ij}^t \; \alpha$$

Train wheel degradation data revisited (cont.)

Where

- $n = 110$ wheels, m_i is the number of measurements per wheel ($i = 1, ..., n$) and $m_i \leq 13$.
- η_i is the log-wear rate of the $i^t h$ sample unit (wheel); it is a function of the working positions and individual unit characteristics.
- β_i is the random effect associated with the i^{th} sample unit; it represents individual unit characteristics.

Train wheel degradation data revisited (cont.)

- $X_i j = [X_{ij1}, X_{ij2}, X_{ij3}, (X_{ij1} \times X_{ij2}), (X_{ij1} \times X_{ij3}), (X_{ij2} \times X_{ij3})]^t$ is a 6×1 vector of covariates associated with Y_{ij}.
- $X_{ijl}(l = 1, 2, 3)$ are dummy variables indicating the working positions **side**(right=1), **truck** (back=1) and **axle** (inner=1).
- $X_{ijl} \times X_{ijl^*}(l, l^* = 1, 2, 3, l \neq l^*)$ are dummy variables indicating the three second-order interactions $(side \times truck, side \times axle, truck \times axle)$.

Train wheel degradation data revisited (cont.)

- $\alpha = (\alpha_1, \alpha_2, \alpha_3, \alpha_{12}, \alpha 13, \alpha 23)^t$ is a 6×1 vector of fixed effects. The first three components, namely $\alpha_1, \alpha_2, \alpha_3$, represent the (population) main effects of side, truck and axle, respectively. The other three are associated with the second-order interactions *side* $\times$ *truck*, *side* $\times$ *axle* and *truck* $\times$ *axle*, respectively.
- α_0 is the intercept, corresponding to the mean initial degradation level of the wheel.
- ε_{ij} is the associated random error for unit i at time (distance) t_j. Note that we use t_J instead of t_{ij} since for the data set under study, $t_{ij} = t_j$ for all $i = (1, ..., n)$.

Train wheel degradation data revisited (cont.)

Assumptions:

- The random errors ε_{ij} are *i.i.d* $\mathrm{N}(0;\sigma_\varepsilon^2)$; σ_ε^2 fixed and unknown
- $\beta_i \overset{\text{iid}}{\sim} N\left(\mu_\beta, \sigma_\beta^2\right)$; μ_β and σ_β^2 both fixed and unknown
- the random effects β_i are independent of the errors ε_{ij}.

Train wheel degradation data revisited (cont.)

Advantages of this parameterization:

- ✓ the wear rate is always positive as suggested by the profiles (Figure 2).
- ✓ it is possible to obtain the distribution of the time to failure $F(t)$ analytically:

Train wheel degradation data revisited (cont.)

- $\eta = X^t\alpha + \beta \;\Rightarrow\; \eta \sim N\left(X^t\alpha + \mu_\beta; \sigma_\beta^2\right)$
- $e^\eta \sim \text{logn}\left(X^t\alpha + \mu_\beta; \sigma_\beta\right)$
- $T \sim \text{logn}\left(\mu_T; \sigma_T\right)$

  where

$$\mu_T = log(D_f - \alpha_0) - (X^t\alpha + \mu_\beta) \;\text{ and }\; \sigma_T = \sigma_\beta$$

Train wheel degradation data revisited (cont.)

RESULTS

- interaction $AXLE \times SIDE$ significant $(p < 0,02)$

Train wheel degradation data revisited (cont.)

Table : Interval and Point estimates of the reliability figures by working conditions - NMLE

Working condition [SIDE,AXLE]	Estimates ($\times 10^3$ Km)			
	MTTF	$t_{0.01}$	$t_{0.05}$	$t_{0.10}$
[0,0]	1306.6	196.9	314.6	408.6
[L,OUTER]	[979;1.714]*	[142;281]	[233;438]	[307;558]
[0,1]	1187	171.2	258.5	365.8
[L,INNER]	[893;1519]	[132;242]	[218;383]	[284;489]
[1,0]	1057	157.3	254.6	329.4
[R,OUTER	[773;1421,8]	[117;223]	[192;350]	[250;450]
[1,1]=	1573.5	225.1	367.3	477.4
[R,INNER]	[1195;1961]	[169;330]	[279;509]	[367;664]

(*)I.C. Bootstrap 90% (rounded values)

Train wheel degradation data revisited (cont.)

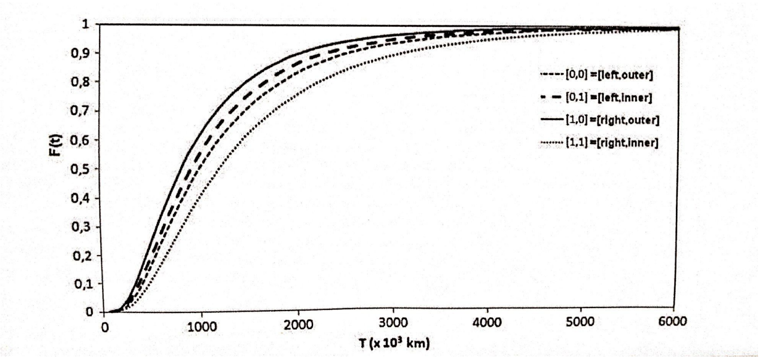

Figure : Fitted time-to-failure distributions $F(t)$ by working condition

Conclusions

- By modeling the unit-specific log-rates ($\eta_i, i = 1, ..., n$) as linear functions of covariates (working positions) and unit-specific normally distributed random effect (β_i; $i = 1, ..., n$), it was possible to investigate possible effects of working conditions (side,axle and truck positions) on the failure time distribution.

- It was possible to identify working positions where the wheels are subject to higher levels of stress and whose difference in wear rate affected the time-to-failure distributions substantially [side=right; axle=outer].

Conclusions

- In addition, since the unit-specific log-rates are linear functions of normally distributed random effects, it was possible to use functions developed for normal random effects, which have already been implemented in a number of softwares such as S-Plus, SAS, R and MATLAB. Here, we used the NLME function available in R.
- Even with the inclusion of the covariates in the log-rate equation, it was still possible to obtain the failure time distributions by working positions analytically, which in turn dramatically reduced the computational effort.