Static and Dynamic Maintenance Policies under Imperfect Repair Models

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Introduction

- Designing appropriate maintenance policies helps to maintain equipments running longer and minimize operation costs.
- Since Barlow and Hunter (1960), much of the literature regarding optimal maintenance of repairable systems has assumed both *minimal repair* (MR) and *perfect preventive maintenance* (PM).
- However, since the MR assumption is too restrictive, attention has been given recently to *imperfect repair* (IR) models (Kijima *et al.*,1988; Doyen & Gaudoin, 2004; Pan & Rigdon, 2009, Corset *et al.*, 2012).

Counting processes

- Let $N(t)$ be the number of failures up to time t
- The random variables $\{T_n : n = 1, 2, \ldots\}$ are the *jump* (or *failure times*) of $N(t)$
- The distribution of $\{N(t)\}_{t \geq 0}$ can be defined by the (conditional) intensity function

$$\lambda(t) = \lim_{h \to 0} \frac{P(N(t+h) - N(t) = 1 | \Im_t)}{h}, \quad \forall t \geq 0$$

where $\Im_t$ is, informally, the history up to time t (the event $\{T_1 = t_1, \ldots, T_{N(t)} = t_{N(t)}\}$).

- The cumulative intensity is $\Lambda(t) = \int_0^t \lambda_u(du)$.
- The *mean function* of the process is $\Phi(t) = E[\Lambda(t)]$ and the *rate of occurrence of failures* (ROCOF) function is $\phi(t) = \Phi'(t) = E[\lambda(t)]$.

  In a sense, the ROCOF function is an unconditional version of the intensity $\lambda(t)$.

  For an NHPP, for which $\lambda(t)$ is deterministic, the intensity and ROCOF functions coincide.

Imperfect repair (IR) models

IR models are specified as a counting process which

1. starts at $t = 0$ with a reference or base deterministic intensity $\lambda_R(t)$ and
2. has a rule which determines how the intensity changes immediately after a failure.

For instance, *virtual age* (VA) models start with $\lambda_R(t)$ and define an associated process $V(t)$ (the virtual age) whereby the intensity is $\lambda(t) = \lambda_R[V(t)]$.

Special cases are $V(t) = t - (1 - \theta)t_{N(t)}$ (Kijima *et al.*, 1988), where θ is a parameter associated to the efficacy of the repair: $\theta = 1$ implies that the repair is minimal; $\theta = 0$ implies that the repair becomes perfect.

Doyen and Gaudoin (2004) define a whole class of VA models by letting

$$V(t) = t - (1-\theta) \sum_{j=0}^{min(m-1,N(t)-1)} \theta^j T_{N(t)-j}.$$

They call this the *arithmetic reduction of age with memory m* (ARA_m) model (Kijima *et al.* (1988) is the ARA_1 model).

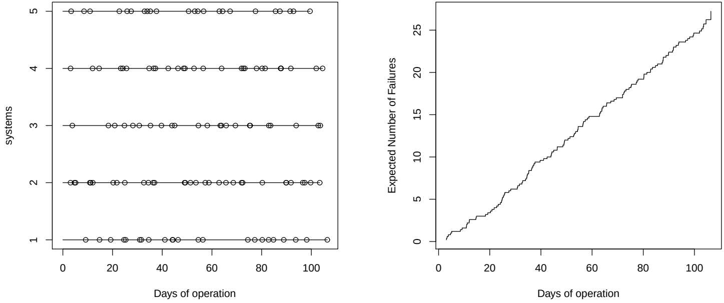
They also defined the *arithmetic reduction of intensity with memory m* (ARA_m) model as

$$\lambda(t) = \lambda_R(t) - (1-\theta) \sum_{j=0}^{min(m-1,N(t)-1)} \theta^j \lambda_R(T_{N(t)-j}).$$

Dump trucks data set

- Designed to operate in road highways.
- In mining companies, particularly, they are used under much more severe conditions.

- Data were collected from July to October/2012, for a sample of five trucks
- The cumulative number of days of operation to each failure was registered
- 129 failures were observed, each one followed by a repair
- Failure truncations



(a) (b)

Figure: (a) Failure times (in days of operation) for each truck; (b) Mean cumulative number of failures.

The goal is to provide reliability information to base the decision-making process related to PM policies in the mining company.

Off-road engines data set

- Off-road trucks: transport loose materials such as minerals and waste in mining operations.
- Good performance of this equipment is essential for the financial health of this kind of business.

- The problem refers only to diesel engines, which can be removed from a truck and replaced by another for carrying out maintenance.
- PM actions are implemented in the engines - *reconditioning* - consisting of a full overhaul (AGAN).
- Corrective maintenance has higher cost (on average approximately 23% larger) than PM.
- Extra engines are necessary to act as backups - a maintenance policy is necessary in order to minimize failures and the number of required spare engines, by increasing the fleet availability.

- Failure and PM times were registered for 193 engines
- 208 failures and 50 PM actions
- 52 engines were time truncated

Number of failures	Number of systems
0	51
1	88
2	43
3	10
4	1
Total	193

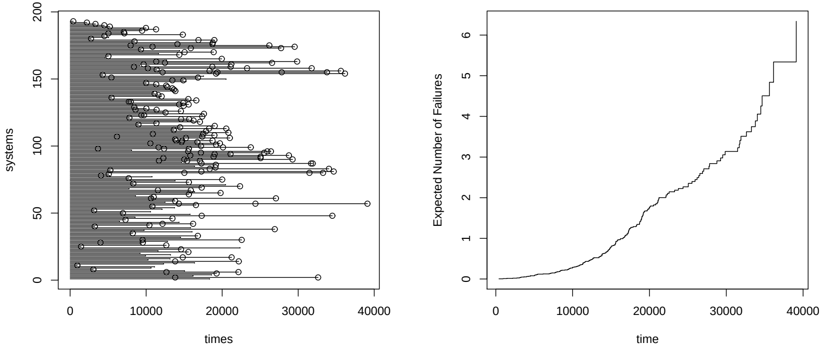


Figure: (a) Failure timed (in hours of operation) for each engine; (b) Mean cumulative number of failures.

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ARA and ARI imperfect repair models: Estimation, goodness-of-fit and reliability prediction

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ABSTRACT

An appropriate maintenance policy is essential to reduce expenses and risks related to equipment failures. A fundamental aspect to be considered when specifying such policies is to be able to predict the reliability of the systems under study, based on a well fitted model. In this paper, the classes of models Arithmetic Reduction of Age and Arithmetic Reduction of Intensity are explored. Likelihood functions for such models are derived, and a graphical method is proposed for model selection. A real data set involving failures in trucks used by a Brazilian mining is analyzed considering models with different memories. Parameters, namely, shape and scale for Power Law Process, and the efficiency of repair were estimated for the best fitted model. Estimation of model parameters allowed us to derive reliability estimators to predict the behavior of the failure process. These results are a valuable information for the mining company and can be used to support decision making regarding preventive maintenance policy.
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Likelihood functions for ARA and ARI models

Notation and assumptions:

- k identical repairable systems, where the failures history occurs independently;
- at each failure, a repair action of degree θ is performed;
- n_i failures are observed in the i^{th} system, $i = 1, 2, \ldots, k$;
- $N = \sum_{i=1}^{k} n_i$ is the total number of observed failures in the systems.

Likelihood function for ARA class of models

$$
\begin{aligned}
&L_{ARA_m}(\mu) = \\
&= \prod_{i=1}^{k} \prod_{j=1}^{n_i} \{\lambda(t_{i,j} - (1-\theta) \sum_{p=0}^{min(m-1,j-2)} \theta^p t_{i,j-1-p}) \times \\
&\times e^{-\Lambda(t_{i,j} - (1-\theta) \sum_{p=0}^{min(m-1,j-2)} \theta^p t_{i,j-1-p}) + \Lambda(t_{i,j-1} - (1-\theta) \sum_{p=0}^{min(m-1,j-2)} \theta^p t_{i,j-1-p})}\} \times \\
&\times e^{-\Lambda(t_i^* - (1-\theta) \sum_{p=0}^{min(m-1,n_i-1)} \theta^p t_{i,n_i-p}) + \Lambda(t_{i,n_i} - (1-\theta) \sum_{p=0}^{min(m-1,n_i-1)} \theta^p t_{i,n_i-p})}
\end{aligned}
$$

If the system is failure truncated, just replace $t_i^* = t_{i,n_i}$.

Likelihood function for ARI class of models

$$L_{ARI_m}(\mu) =$$

$$= \prod_{i=1}^{k} \prod_{j=1}^{n_i} \{[\lambda(t_{i,j}) - (1-\theta) \sum_{p=0}^{min(m-1,j-2)} \theta^p \lambda(t_{i,j-1-p})] \times$$

$$\times e^{-\Lambda(t_{i,j})+\Lambda(t_{i,j-1})+(1-\theta)[t_{i,j}-t_{i,j-1}]\sum_{p=0}^{min(m-1,j-2)} \theta^p \lambda(t_{i,j-1-p})}\} \times$$

$$\times e^{-\Lambda(t_i^*)+\Lambda(t_{i,n_i})+(1-\theta)[t_i^*-t_{i,n_i}]\sum_{p=0}^{min(m-1,n_i-1)} \theta^p \lambda(t_{i,n_i-p})},$$

If the system is failure truncated, just replace $t_i^* = t_{i,n_i}$.

Parameter estimation in ARA and ARI models

For both cases, the likelihood function was rewritten assuming a PLP parametric form for the initial intensity :

$$\lambda(t) = \frac{\beta}{\eta} \left\{ \frac{t}{\eta} \right\}^{\beta-1}$$

therefore, $\mu = (\beta; \eta; \theta)$ is the vector of parameters to be estimated.

Reliability prediction functions - (ARI_m model).

$$
\begin{aligned}
R_{T_n, ARI_m}(t) &= P(T_{n+1} - T_n > t | \Im_{t_n}) \\
&= exp\left\{ \left(\frac{t_n}{\eta}\right)^{\beta} - \left(\frac{t_n + t}{\eta}\right)^{\beta} \right\} \times \\
\times \quad & exp\left\{ t(1-\theta) \sum_{j=0}^{Min(m-1,n)-1} \theta^j \frac{\beta}{\eta} \left(\frac{t_{n-j}}{\eta}\right)^{\beta-1} \right\}.
\end{aligned}
$$

Reliability prediction functions - (ARA_m model).

$$
\begin{aligned}
R_{T_n, ARA_m}(t) &= exp\left\{-\left(\frac{t-(1-\theta)\sum_{j=1}^{Min(m-1,n-1)}\theta^j t_{n-j}}{\eta}\right)^{\beta}\right\} \times \\
&\times \; exp\left\{\left(\frac{-(1-\theta)\sum_{j=1}^{Min(m-1,n-1)}\theta^j t_{n-j}}{\eta}\right)^{\beta}\right\} \qquad (1)
\end{aligned}
$$

Model selection based on the log-likelihood

- maximum value of the estimated likelihoods: $\hat{L} = L(\hat{\theta}; \hat{\beta}; \hat{\eta})$

- Burnham and Anderson (2004) : weight of evidence in favor of model r, given by:

$$w_r = \frac{exp(-\Delta_r/2)}{\sum_{r=1}^{R} exp(-\Delta_r/2)}$$

where

- $\Delta_r = \hat{L}_{max} - \hat{L}_r, \;\; (r = 1, \ldots, R)$

- and $\hat{L}_{max}$ is the maximum of the R different $\hat{L}$ values, considering that R different models were fitted.

This transformation forces the best model to have $\Delta = 0$, while the rest of the models have positive values.

Model selection based on a goodness-of-fit plot

- The *mean function* $\Phi(.)$ is estimated to each model according to these steps:
  - ML estimates are obtained for a model from its observed failure data.
  - Observed failure data for the i-th system $(i = 1, \ldots, k)$ and the MLEs are plugged in the model intensity function, providing $\hat{\lambda}_i(t)$.
  - $\hat{\Phi}_i(t)$ is computed as $\int_0^t \hat{\lambda}_i(u)du$, for $0 \leq t \leq t_{i,n_i}$ if the i-th system is truncated with n_i failures, and $0 \leq t \leq t_i^*$ if the i-th system is truncated at time t_i^*.
  - Finally, $\hat{\Phi}(t)$ is obtained as $\frac{\sum_{i=1}^{k} I_i \hat{\Phi}_i(t)}{n_t}$, where I_i is the indicator for the i-th system being at risk at time t, and n_t is the number of systems at risk at time t.
- Plot of $\hat{\Phi}(t)$ against empirical MCF (Nelson-Aalen plot, or mean cumulative number of failures plot).

Dump trucks data set revisited

The following models were considered:

1. $ARA_m; \;\; m = 1, \ldots, 31$
2. $ARI_m; \;\; m = 1, \ldots, 31$
3. Minimal Repair ($\theta = 1$)

In cases (1) and (2), $m = 31$ corresponds to $m = \infty$ (max number of failures) and the PLP parametric form for the initial intensity was adopted.

Table: Results of the model fitting - Dump trucks data

Estimated values	MODELS			
	MR	ARA_1	ARA_{13}	ARA_∞
$\hat{\beta}$	1.14	1.33	1.80	1.81
	[0.96;1.35]	[1.05;1.69]	[1.39;2.35]	[1.39;2.35]
$\hat{\eta}$	5.92	4.94	7.58	7.59
	[3.54;9.93]	[3.41;7.15]	[5.35;10.76]	[5.35;10.79]
$\hat{\theta}$	-	0.02	0.60	0.60
	-	[0.0001;0.53]	[0.43;0.84]	[0.42;0.84]
$\hat{L}$	-307.1811	-304.7039	-300.3218	-300.3165
		ARI_1	ARI_{13}	ARI_∞
$\hat{\beta}$		1.42	1.89	1.90
		[1.06;1.91]	[1.69;2.11]	[1.71;2.11]
$\hat{\eta}$		4.18	7.48	7.65
		[2.57; 6.79]	[5.74; 10.11]	[5.76;10.17]
$\hat{\theta}$		0.23	0.67	0.67
		[0.05;1.00]	[0.52;0.87]	[0.52;0.87]
$\hat{L}$		-306.2146	-300.0904	-300.1155

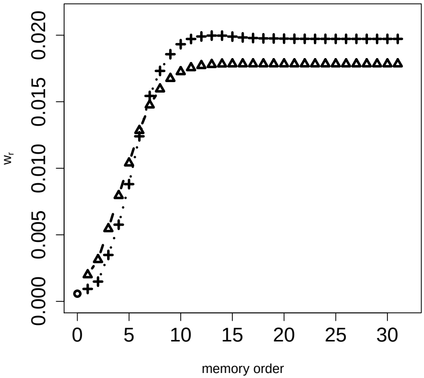
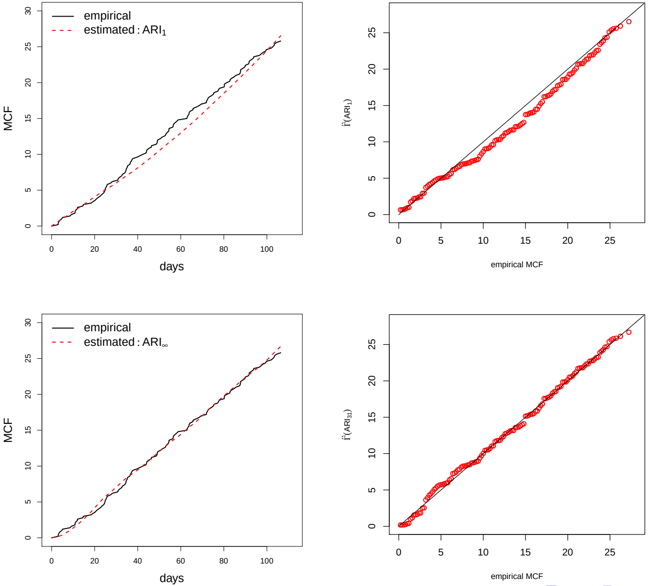


Figure: Criterion value for model selection (0 - MR, + ARI and Δ - ARA).


empirical
estimated : ARI$_1$
MCF
days
empirical MCF
empirical
estimated : ARI$_\infty$
MCF
days
empirical MCF

- for ARI_∞, $\hat{\beta} = 1.90$ ([1.71; 2.11]) , indicating that the equipment failure intensity function increases with time (intrinsic aging)
- $\hat{\theta} = 0.67$ ([0.52,0.87]). The repairs after failures tend to leave the equipment in a state between **AGAN and ABAO**

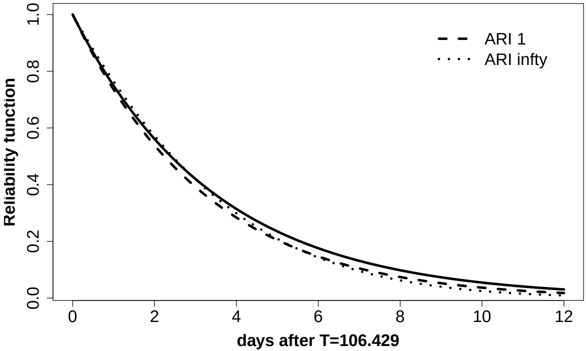


Figure: Estimated reliability functions at $T_{23} = 106.429$ days for the trucks data set under the fitted models.

- Imperfect Repair Classes of Models (ARA and ARI).
- Inference for these classes of models (Estimation, Models Selection and Reliability Predictors).
- Illustration in a real data set related to mining dump trucks.
- Further development of goodness of fit procedures: graphical procedure.
- For the dump trucks data: effect of repair, aging speed and reliability predictor for maintenance policy.

Cost minimization problem under MR

For a repairable system that is put in operation at time $t = 0$, assume:

- PM check points are scheduled after every τ units of time;
- at each PM, a (perfect) maintenance of expected cost C_M takes place;
- between successive PM check points, a repair with expected cost C_R is done;
- Let $C(\tau) = C_M + C_R N(\tau)$ be the total cost between two successive PMs performed τ units of time apart and $H(\tau) = C(\tau)/\tau$ be the corresponding cost *per unit of time*.

Since for an NHPP we have that $E[N(t)] = \Lambda(t) = \int_0^t \lambda(u)du$, the expected cost per unit of time is $E[H(\tau)] = [C_M + C_R\Lambda(\tau)]/\tau$.

Deriving and equating to zero we obtain that the optimal period τ_P must be the solution of

$$B(\tau) := \tau\lambda(\tau) - \Lambda(\tau) = \frac{C_M}{C_R}, \tag{2}$$

or $\tau_P = B^{-1}(C_M/C_R)$, where $\lambda(t)$ is increasing.

This is the solution of Barlow & Hunter (1960). Further developments can be found among others in Glasser (1967); Bassin (1973); Gilardoni & Colosimo (2007, 2011) and Gilardoni, Oliveira and Colosimo (2013). This optimal policy is a *periodical age-dependent policy*.

Cost minimization problem under IR

This maintenance policy can be extended for IR models since, from a purely mathematical point of view, an IR model is a general counting process with a random intensity $\lambda(t)$.

Now, the expected cost per unit of time becomes

$$E[H(\tau)] = [C_M + C_R\Phi(\tau)]/\tau,$$

and, hence, the optimal solution is $\tau_P = B^{-1}(C_M/C_R)$, where now

$$B(\tau) := \tau\phi(\tau) - \Phi(\tau) = \frac{C_M}{C_R} \tag{3}$$

(compare with (2)).

For ARA_1 model (Doyen & Gaudoin, 2004), we have

$$\lambda(t) = \lambda_R(t - (1-\theta)T_{N(t)}). \tag{4}$$

So, the mean function of the process is given by

$$\Phi(\tau) = \int_0^{\tau} E[\lambda_R(t - (1-\theta)T_{N(t)})]dt. \tag{5}$$

So, there is no closed form solution for this equation (since $T_{N(t)}$ is a random variable).

Although some approximations have been proposed (Kijima *et al.* 1988; Yevkin & Krivtsov, 2012), our interest is to estimate $\Phi(t)$ from data.

Estimation of the optimal periodic policy

- **Step 1: Maximum Likelihood estimation of the model parameters.** Use the failure history and log-likelihood function to obtain the MLEs $\hat{\beta}$, $\hat{\eta}$ and $\hat{\theta}$.
- **Step 2: Estimation of the mean function $\Phi(t)$:** Monte Carlo simulation of failure histories and calculation of the MCF.
  - **Step 2.1: Monte Carlo simulation.** Use the estimated values $\hat{\beta}$, $\hat{\eta}$ and $\hat{\theta}$ to generate K failure processes (K large) truncated in time T.
  - **Step 2.2: Calculation of the MCF $\hat{\Phi}(t)$.** As the failure processes generated in Step 2.1 are time truncated, the MCF is simply the mean number of observed failures between 0 and T on the K trajectories.

- **Step 3: Estimation of the optimal periodicity τ.** In order to solve Equation (3), it is necessary to find estimates for the functions $\phi(t)$ and $\Phi(t)$.

  In Step 2, the MCF was used as an estimate for $\Phi(t)$. However, the MCF is a step function. So, we use here the nonparametric estimate given by the right derivative of the Greatest Convex Minorant (GCM) (Boswell, 1966):

  1. The GCM of $\hat{\Phi}(t)$, namely $\hat{\Phi}_{S_G}(t)$, is given by:

  $$\hat{\Phi}_{S_G}(t) = sup\{g(t) : g \text{ is convex and } g(u) \leq \hat{\Phi}(u) \text{ for all } u\}; \qquad (6)$$

  2. Then, $\hat{\phi}_{S_G}(t) = \hat{\Phi}'_{S_G}(t+0)$ (the right derivative of $\hat{\Phi}_{S_G}(t)$).

Appropriate confidence intervals for τ are obtained using nonparametric bootstrap resampling (Efron and Tibshirani, 1986).

Off-road engines data set revisited

Table: Point and interval (95% confidence level) estimates for PLP (β, η) and effect of repair (θ) parameters, and values of the maximum of the log-likelihood function ($\hat{l}$), AIC e BIC under minimal and imperfect repair models for off-road data.

Model:	Minimal Repair	Imperfect Repair (ARA_1)
$\hat{\beta}$	2.125(1.916; 2.357)	2.458(2.185; 2.765)
$\hat{\eta}$	16,715(15,604; 17,905)	15,582(14,601; 16,628)
$\hat{\theta}$	-	0.471(0.330; 0.672)
$\hat{l}$	-2126.74	-2118.59
AIC	4257.48	4243.18
BIC	4264.15	4253.19

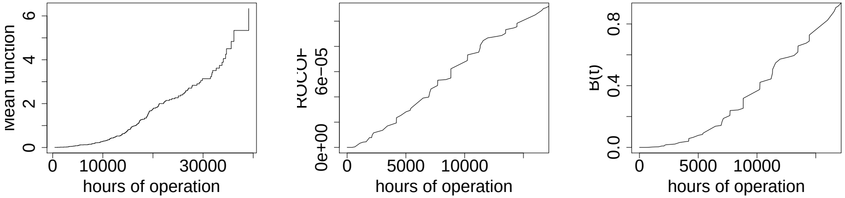


Figure: Estimated functions for the engine data set: (a) Mean cumulative number of failures, (b) $\hat{\phi}(t)$ and (c) $\hat{B}(t)$, versus t (in hours of operation).

Table: Off road engines data – Optimal Periodic Maintenance Policy by Cost Ratio (C_{PM}/C_{IR}), under MR ($\hat{\tau}_{MR}$) and IR ($\hat{\tau}_{IR}$) assumptions, and bootstrap (B=10,000) confidence intervals

C_{PM}/C_{IR}	$\hat{\tau}_{MR}$	95% CI	$\hat{\tau}_{IR}$	95% CI
1/1.23	14,345	(13,304; 15,511)	15,815	(13,632; 18,082)
1/3	9,429	(8,898; 9,995)	9,207	(8,608; 10,173)
1/5	7,414	(6,974; 7,901)	7,500	(6,720; 8,125)
1/10	5,350	(4,949; 5,820)	5,593	(4,847; 6,227)
1/15	4,421	(4,028; 4,888)	4,621	(4,017; 5,386)

As far as the practical results are concerned, two important pieces of information were provided to the mining company:

- the degree of repair: the estimated value of θ and the corresponding confidence interval indicated that the repair actions after failures are neither minimal nor perfect repairs.
- optimal preventive maintenance check points: in order to minimize the total expected maintenance cost, the mining company needs to implement PM after every $15,815$ h, i.e., around 22 months, with confidence level of 95%.

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Innovative Applications of O.R.

Dynamics of an optimal maintenance policy for imperfect repair models

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ABSTRACT

A preventive maintenance policy that considers information provided by observing the failure history of a repairable system is proposed. For a system that is to be operated for a long time, it is shown that the proposed policy will have a lower expected cost than a periodical one which does not take into account the failure history. Statistical inference using both maximum likelihood point estimates and bootstrap confidence intervals is discussed. The proposed policy is applied to a real situation involving maintenance of off-road engines owned by a Brazilian mining company. A simulation study compares the performance between the maintenance policy proposed and the periodical one.

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- When one has access to the failure history of the equipment when deciding the maintenance policy, the previous solution (τ_P) is not optimal for IR models.

- From a mathematical viewpoint, the reason is that, since the resulting process $N(t)$ will not have independent increments, the failure history provides information about the future reliability of the system.

- Consequently, it should be taken into account when determining the optimal policy.

- Hence, the objective is to understand how the information provided by the failure history of the system can be used to determine maintenance policies for IR models which improve the optimal periodical policy.

The dynamic approach

If we consider a process where new information is becoming available continuously over time and the corresponding PM policy is being revised accordingly, a PM will occur if and only if

$$\lambda(t) = \phi[B^{-1}(C_M/C_R)] \tag{7}$$

For a virtual age model with $\lambda(t) = \lambda_R[V(t)]$, (7) becomes

$$V(t) = \lambda_R^{-1}\{\phi[B^{-1}(C_M/C_R)]\} := \tau_{VA}, \tag{8}$$

meaning that the system will be maintained whenever the virtual age reaches a threshold (i.e., the dynamic policy is periodical in the virtual age scale).

- Implementation of the *dynamic* policy (7) is straightforward: one needs to compute (an estimate of) $\phi[B^{-1}(C_M/C_R)]$ and check it against (an estimate of) the intensity of the process.

- For instance, for ARA_1 model (Doyen & Gaudoin, 2004), we have $\lambda(t) = \lambda_R[t - (1-\theta)t_{N(t)}]$ and hence (7) becomes

$$\lambda_R[t - (1-\theta)t_{N(t)}] = \phi[B^{-1}(C_M/C_R)] \tag{9}$$

  or, equivalently,

$$t - t_{N(t)} = \theta t_{N(t)} + \lambda_R^{-1}\{\phi[B^{-1}(C_M/C_R)]\}. \tag{10}$$

In other words, at each failure the operator computes the right hand side of (10). This is the waiting lapse until the next PM provided that no new failure occur before it. On the contrary, if a new failure occurs before the PM is performed, (10) is computed again to obtain a new PM lapse and so on.

Statistical inference

The estimates $\hat{\Phi}(t)$ and $\hat{\phi}(t)$, as described before, can now be used to compute

$$\hat{B}(t) = t\hat{\phi}(t) - \hat{\Phi}(t)\,. \tag{11}$$

Inverting $\hat{B}(t)$ we obtain an estimate of $\tau_P = B^{-1}(C_M/C_R)$.

Likewise, the right hand side of (10) can be estimated now after noting that, for the PLP reference intensity, $\lambda_R^{-1}(x) = \eta[\eta x/\beta]^{1/(\beta-1)}$.

Since the right hand side of (10) does not depend on the history of the system, the Monte Carlo simulation has to be done only once during the entire process.

As $\Phi(t)$ and $\phi(t)$ are estimated using an auxiliary Monte Carlo simulation, the calculation of standard deviations using the Delta Method is difficult.

A bootstrap sample of τ_{VA} can be obtained as follows:

1. Use the MLEs $\hat{\beta}$, $\hat{\eta}$ and $\hat{\theta}$ to generate K systems under the IR model desired with the same truncation structure as the original data set;
2. Use the generated data set to compute the MLEs $\hat{\beta}^{(b)}$, $\hat{\eta}^{(b)}$ and $\hat{\theta}^{(b)}$;
3. Use the Monte Carlo simulation described before to compute estimates $\hat{\Phi}^{(b)}(t)$, $\hat{\phi}^{(b)}(t)$ and then $(\hat{B}^{-1})^{(b)}(t)$;
4. Given a cost ratio C_M/C_R, $\tau_{VA}^{(b)}$ is obtained from (8).
5. Repeat the procedure above B times to obtain $(\hat{\tau}_{VA}^{(1)}, \ldots, \hat{\tau}_{VA}^{(B)})$. The sample percentiles can then be used to construct the desired confidence interval for τ_{VA}.

Comparison with the periodical policy

- A simulation study compared $E[C(\tau_P)]$ and $E[C(\tau_D)]$, using R.
- For each possible scenario, failure times were generated for $N = 100,000$ systems according to an ARA_1 model with a PLP reference intensity.
- The 120 scenarios were defined by combining
  - $\beta = 1.5$, 2.0, 2.5 and 3,
  - $\theta = 0.1$, 0.3, 0.5, 0.7 and $0,9$,
  - $\frac{C_M}{C_R} = \frac{1}{3}$, $\frac{1}{5}$ and $\frac{1}{15}$ and
  - both the periodic and the dynamic policies.

The η value remained fixed at $15,000$ units of time.

Table: Mean cost per unit time point estimates[1] and standard errors[2] for the periodic and dynamic policies

		$C_M/C_R = 1/3$		$C_M/C_R = 1/5$		$C_M/C_R = 1/15$	
β	θ	periodic	Dynamic	periodic	Dynamic	periodic	Dynamic
1.5	0.1	2.39 (4.41)	2.00 (4.10)	3.50 (10.7)	2.80 (7.69)	7.48 (42.3)	5.96 (29.5)
2.0	0.1	2.17 (5.03)	1.80 (3.34)	2.89 (9.61)	2.42 (6.03)	5.18 (30.3)	4.21 (19.6)
2.5	0.1	1.95 (4.78)	1.70 (2.63)	2.43 (8.02)	2.06 (4.65)	3.85 (22.9)	3.35 (13.6)
3.0	0.1	1.78 (4.29)	1.51 (2.23)	2.12 (6.75)	1.86 (3.63)	3.12 (18.2)	2.70 (10.3)
1.5	0.3	2.48 (5.51)	2.10 (4.71)	3.57 (10.9)	3.02 (8.87)	7.62 (43.4)	6.34 (32.6)
2.0	0.3	2.23 (5.44)	1.90 (3.94)	2.91 (9.71)	2.46 (6.81)	5.12 (30.6)	4.41 (21.2)
2.5	0.3	1.96 (4.88)	1.72 (3.17)	2.44 (8.12)	2.17 (5.24)	3.82 (22.8)	3.38 (15.2)
3.0	0.3	1.77 (4.34)	1.57 (2.62)	2.11 (6.69)	1.95 (4.14)	3.10 (17.7)	2.81 (11.3)
1.5	0.5	2.53 (5.93)	2.25 (5.36)	3.59 (11.6)	3.17 (9.87)	7.65 (42.2)	6.59 (35.4)
2.0	0.5	2.24 (5.78)	2.01 (4.62)	2.91 (9.97)	2.60 (7.80)	5.11 (30.9)	4.52 (23.4)
2.5	0.5	1.98 (5.07)	1.79 (3.78)	2.46 (8.17)	2.21 (6.18)	3.86 (23.0)	3.45 (17.1)
3.0	0.5	1.78 (4.34)	1.64 (3.09)	2.13 (6.95)	1.94 (4.99)	3.13 (16.6)	2.82 (12.7)
1.5	0.7	2.58 (6.51)	2.39 (5.99)	3.65 (12.0)	3.34 (10.7)	7.74 (46.0)	7.04 (39.6)
2.0	0.7	2.29 (6.02)	2.11 (5.27)	2.96 (10.2)	2.73 (8.83)	5.26 (32.0)	4.72 (26.3)
2.5	0.7	1.99 (5.19)	1.87 (4.40)	2.46 (8.46)	2.30 (6.95)	3.83 (22.2)	3.62 (18.6)
3.0	0.7	1.80 (4.47)	1.70 (3.74)	2.14 (7.01)	2.02 (5.77)	3.18 (16.9)	3.00 (13.8)
1.5	0.9	2.60 (6.65)	2.53 (6.50)	3.67 (11.8)	3.55 (11.8)	7.63 (44.0)	7.47 (43.1)
2.0	0.9	2.30 (6.21)	2.25 (6.01)	2.98 (10.5)	2.90 (9.92)	5.18 (31.6)	5.08 (29.9)
2.5	0.9	2.02 (5.25)	1.97 (5.04)	2.49 (8.55)	2.43 (8.06)	3.88 (23.7)	3.74 (21.4)
3.0	0.9	1.82 (4.62)	1.78 (4.27)	2.15 (7.24)	2.10 (6.69)	3.13 (18.6)	3.03 (16.1)

(1) reported values should be multiplied by 10^{-4}; (2) Monte Carlo standard errors, reported values should be multiplied by 10^{-7}

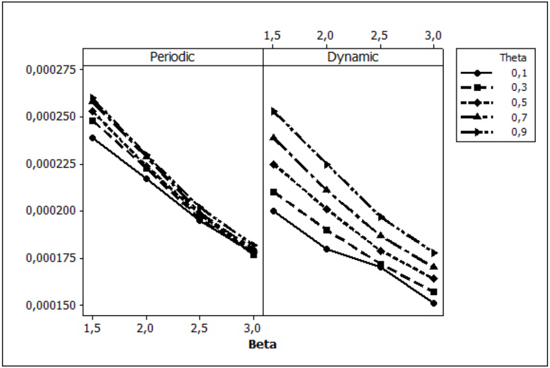

Figure: Estimates for the mean cost per unit time (periodical and dynamic policies) versus β values, by efficiency of repair (θ); CM/CR=1/3

Off-road engines data set revisited

Table: Estimations for the off-road engines data set, under different values of C_M/C_R: Optimal preventive PM period ($\hat{B}^{-1}(C_M/C_R)$); Optimal time for preventive PM based on the dynamic approach ($\hat{\tau}_{VA}$), and bootstrap ($B = 10,000$) 95% confidence intervals (values are in hours)

C_M/C_R	$\hat{B}^{-1}(C_M/C_R)$	95% CI	$\hat{\tau}_{VA}$	95% CI
1/1.23	15,815	(13,632; 18,082)	11,373	(10,978; 12,023)
1/3	9,207	(8,608; 10,173)	8,141	(7,572; 8,647)
1/5	7,500	(6,720; 8,125)	6,537	(6,043; 7,097)
1/10	5,593	(4,847; 6,227)	4,694	(4,403; 5,404)
1/15	4,621	(4,017; 5,386)	4,031	(3,656; 4,596)

- Under a MR PLP model, an estimate of the optimal preventive PM period is $15,815$ hours, with a 95% confidence interval from $13,632$ to $18,082$ hours. For a new system, this is how long the company should wait for the first PM action, no matter how many failures occur until then.
- Under an IR ARA$_1$-PLP model, an estimate of the optimal *virtual time* for the first preventive PM based on the dynamic approach is $\hat{\tau}_{VA} = 11,373$ hours, with a 95% confidence interval from $10,978$ to $12,023$ hours. For a new system, the company should wait until $\hat{V}(t) = t - (1 - \hat{\theta})\, t_{N(t)} = \hat{\tau}_{VA}$ for the first PM action. In practice, it means that the PM action will occur at $t = \hat{\tau}_{VA}$ if no failure occurs until then. If any failure is observed before $\hat{\tau}_{VA} = 11,373$ hours, the optimal time to the first PM must be recalculated according to (10) and so on.

Final remarks

- We proposed a dynamic method to estimate the optimal PM policy for repairable systems.
- This dynamic policy updates the optimal time continuously as the information provided by the failure history of the system becomes available.
- A simulation study showed that the expected operating cost per unit of time may be much lower when using the dynamic policy, especially when either (i) the cost ratio CM/CR is large or (ii) the effect of the repair is far from the MR case.
- The method was applied to a real data set.

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Thank you!

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