



Contribution to deterioration modeling and remaining useful life estimation based on condition monitoring data

PhD defense

Presented by: [Thanh Trung LE](#)

Supervisors: Christophe Bérenguer and Florent Chatelain

Equip SAIGA, GIPSA-lab, Grenoble INP



Thursday, November 26, 2015



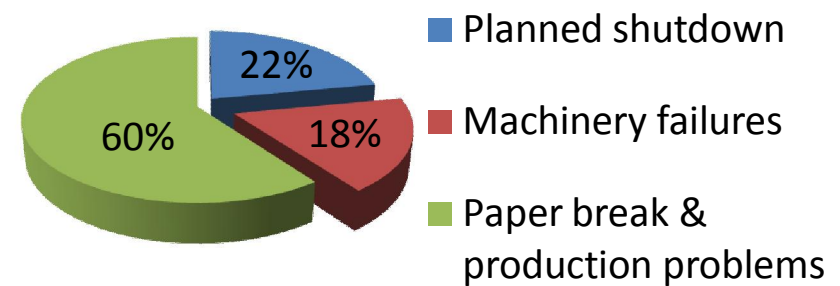
Industrial context

Continuous production systems

- Complex
- Continuity is a critical issue
- Can fail because of a defective component
- Components deteriorate over time

⇒ **Need of advanced maintenance programs**

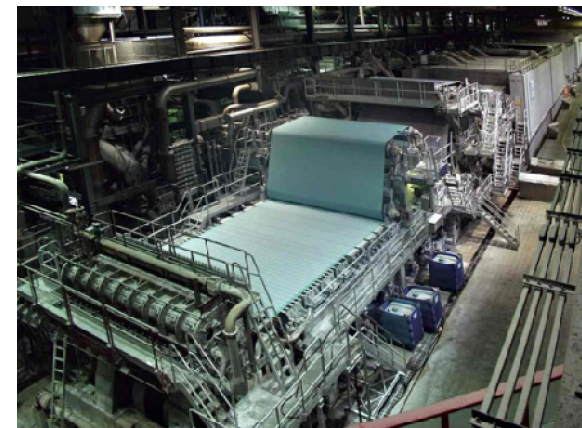
Paper machine down time



Lost ~ **34 m€/year**



Critical components

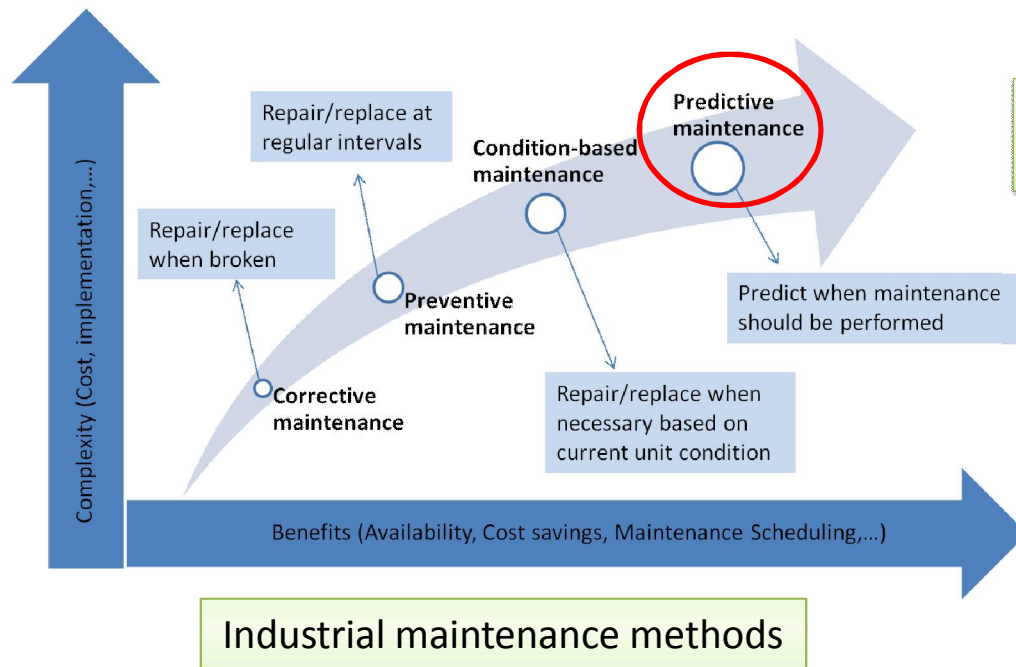


A paper machine

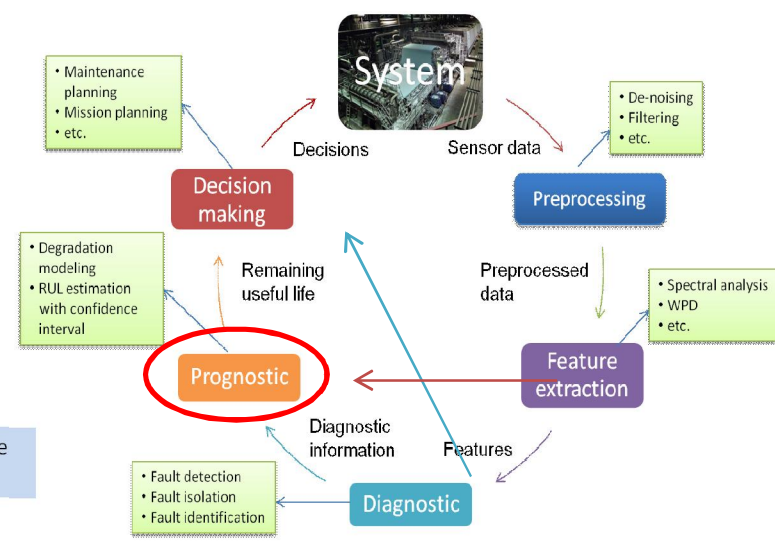
Predictive maintenance

Prognostics

➤ Anticipate the failure



Predictive maintenance program



SUPREME project



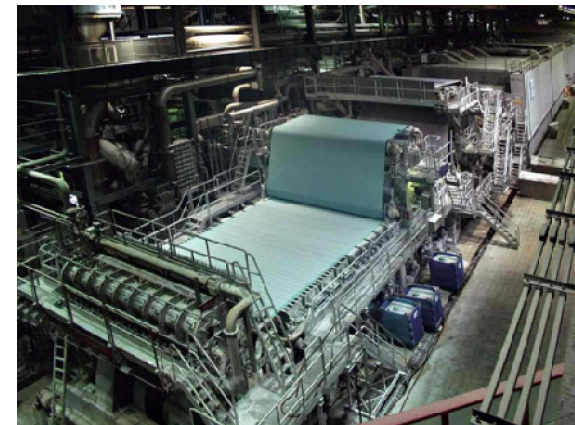
FP7 european project

- “SUstainable PREdictive Maintenance for manufacturing Equipment”
- 10 partners from both industry and academy
- **Purpose:** Development of new tools for predictive maintenance to improve productivity, reduce machine downtimes and increase energy efficiency.

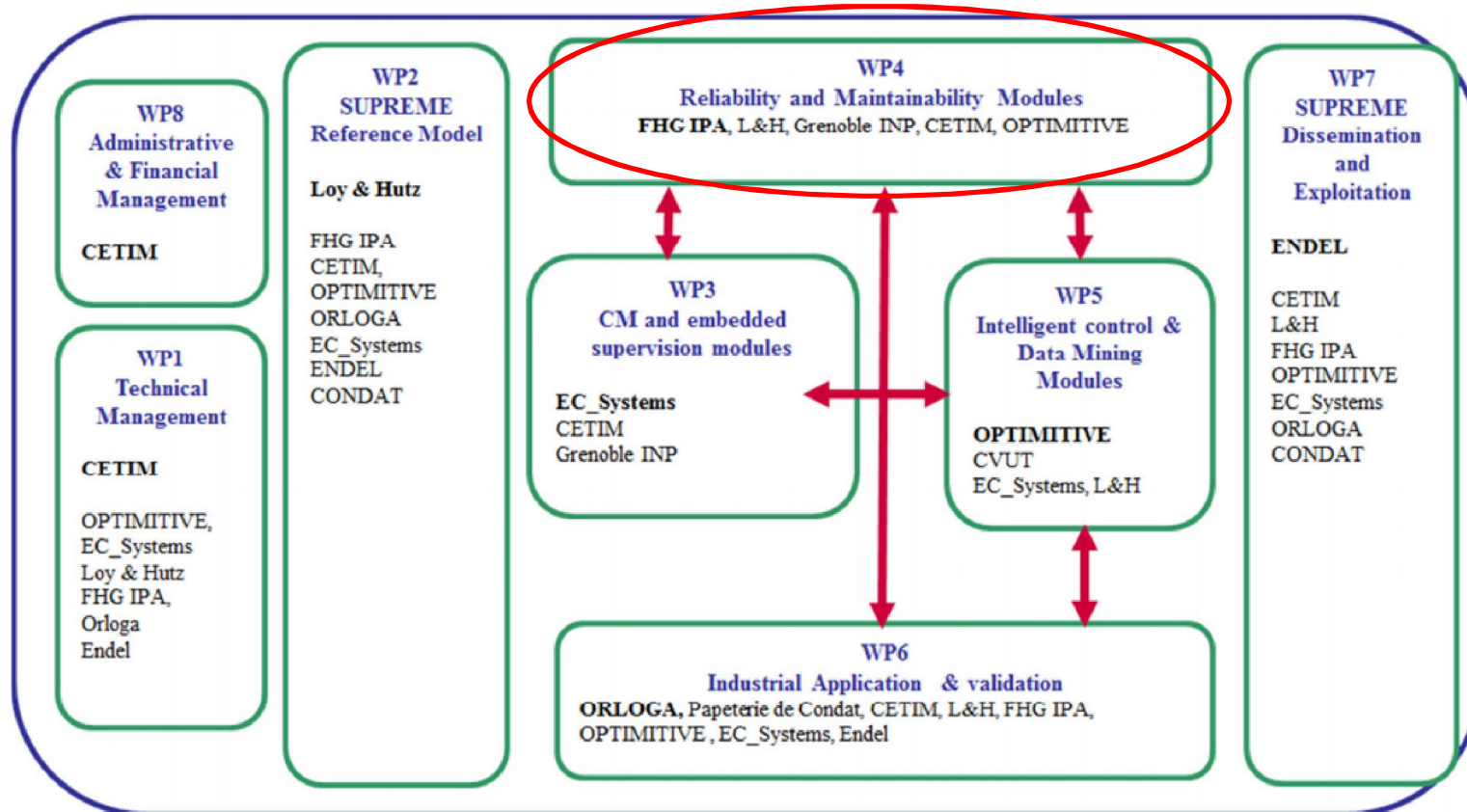
⇒ Application case: Paper machine



SUPREME partners



SUPREME work plan



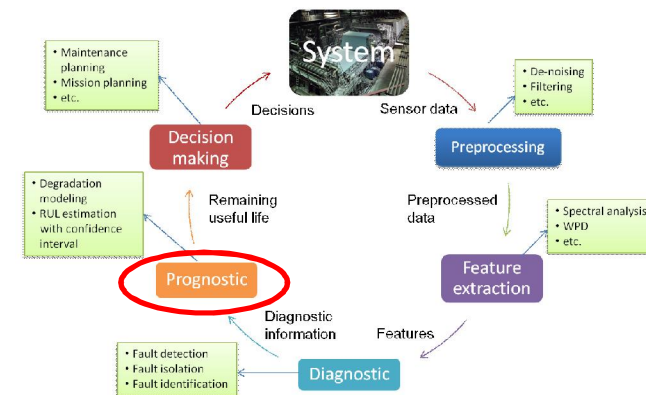
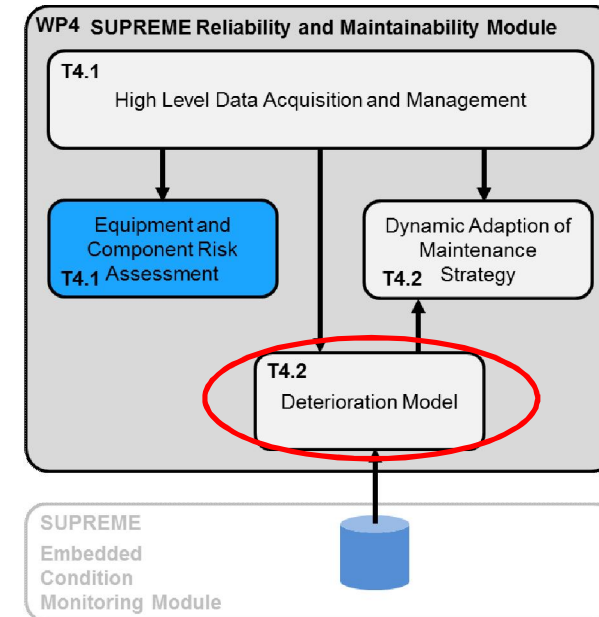
Reliability and maintainability module

Four sub-modules

- Systematic critical component identification based on risk assessment
- Deterioration-based reliability
- Dynamic adaption of maintenance strategy

Objectives of the thesis

- ✓ **Deterioration modeling**
- ✓ **Remaining Useful Life (RUL) estimation**



Outline

- Problem statement
- Multi-branch discrete-state models
- Jump Markov linear systems
- Conclusion & Perspectives

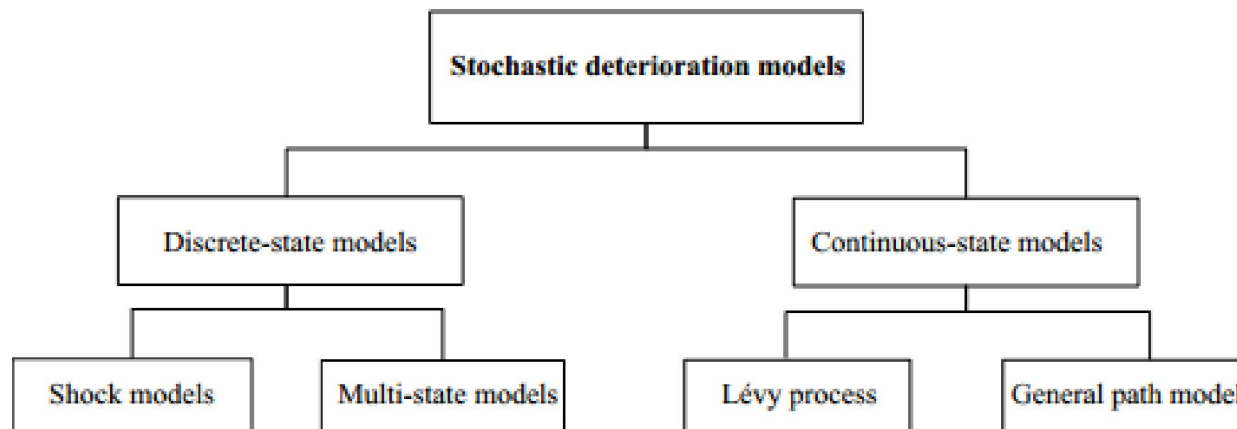
Outline

- Problem statement
- Multi-branch discrete-state models
- Jump Markov linear systems
- Conclusion & Perspectives

Deterioration modeling

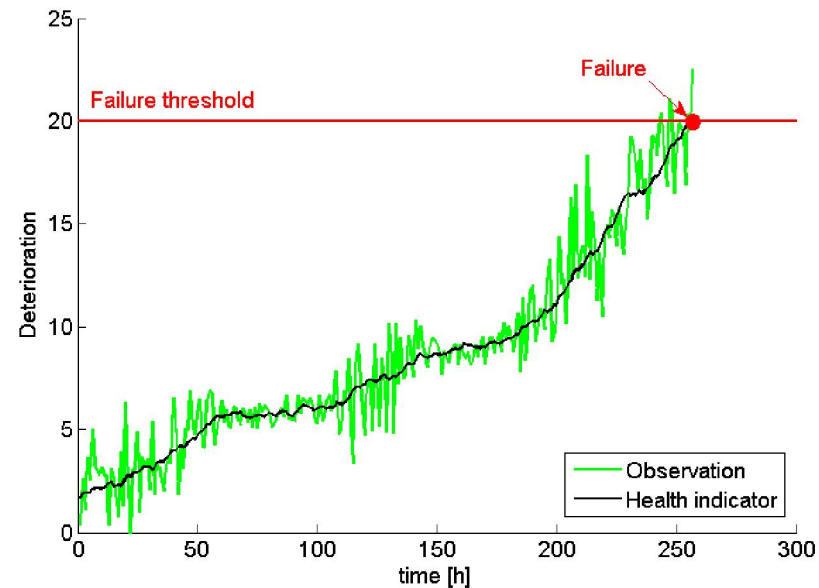
- Model the dynamic stochastic behavior of the deterioration
- Link the failure of an item to its deterioration level
- Allow health state assessment
- Basis for RUL prediction

Classification



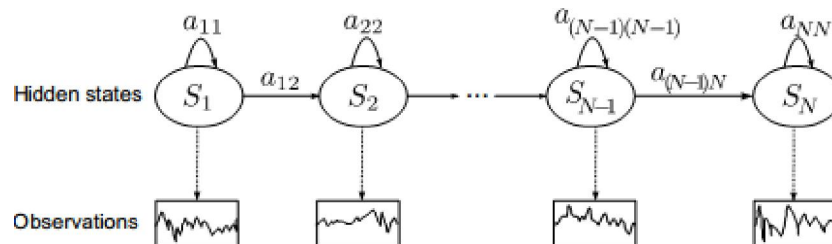
Continuous-state modeling

- Health states are continuous
- Continuous stochastic processes (Gamma, Wiener process,...)
- Stochastic filtering: Kalman, particle filters, etc.

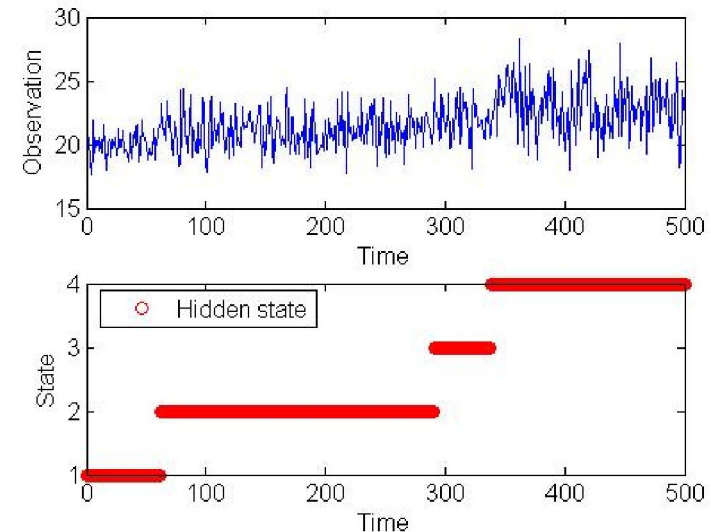


Discrete-state modeling

- Health states are **discrete**: normal, degraded level 1, degraded level 2, failure
- Power tools: Markov-based models
 - Hidden Markov Model (HMM)
 - Hidden semi-Markov Model (HsMM)



An HMM example



Remaining Useful Life (RUL)

- Residual time for accomplishing required functions
- Conditional random variable:

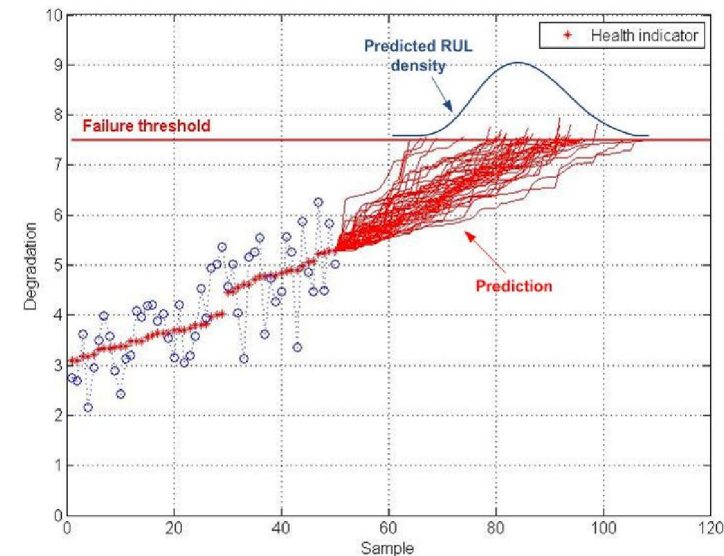
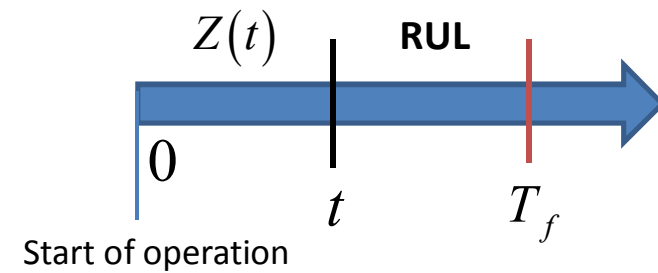
$$RUL = T_f - t \mid T_f > t, Z(t)$$

$Z(t)$: information up to time t ;

T_f : time to failure

- Uncertainties assessment

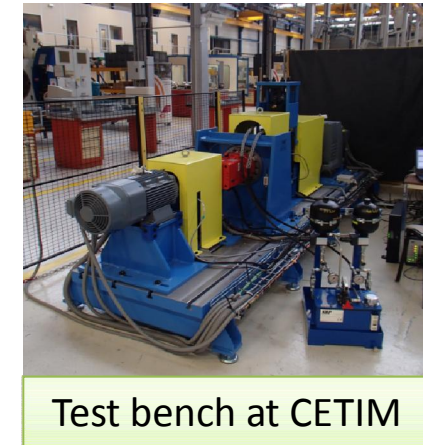
=> characterized by probabilistic distributions



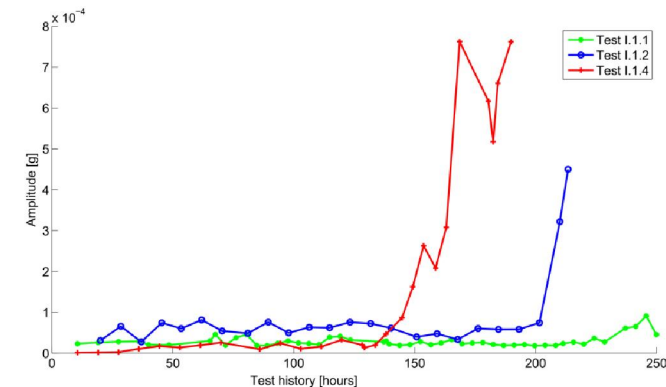
RUL = time to reach critical level

Problems

- CM data may not represent directly deterioration states
 - State-space representation
$$\begin{cases} x_t = f(x_{t-1}, \omega_t) & : \text{Hidden states} \\ y_t = g(x_t, v_t) & : \text{Observations} \end{cases}$$
- Multiple modes co-existence
 - Deterioration rates depend on initiation time
 - Deterioration behavior depend on applied loads



- ⇒ Proposed solution: **Multi-branch modeling**
- **Discrete-state**: Multi-branch Hidden Markov models
 - **Continuous-state**: switching state space model



Bearing health index evolution

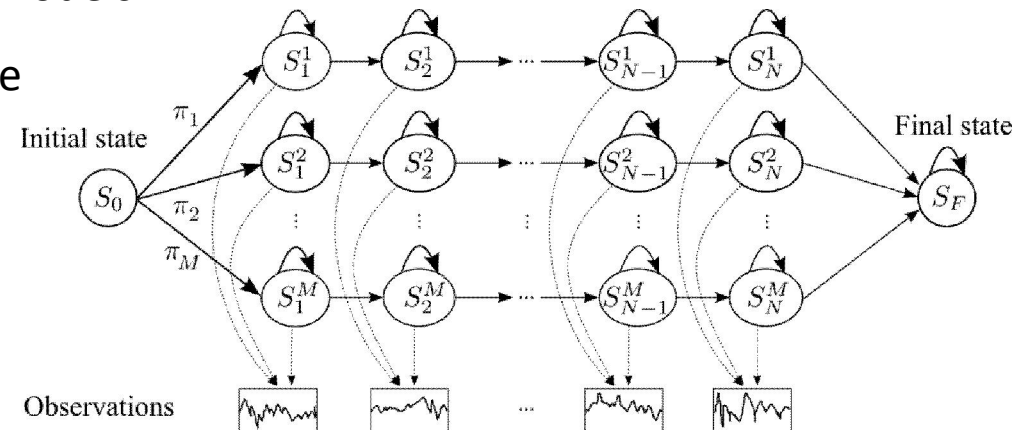
Outline

- Problem statement
- Multi-branch discrete-state models
- Jump Markov linear systems
- Conclusion & Perspectives

Multi-branch discrete-state modeling

Model construction

- Discrete states => Based on Markov models
- M deterioration modes => M branches
- Two common states:
 - Initial: normal condition
 - Final: failure state



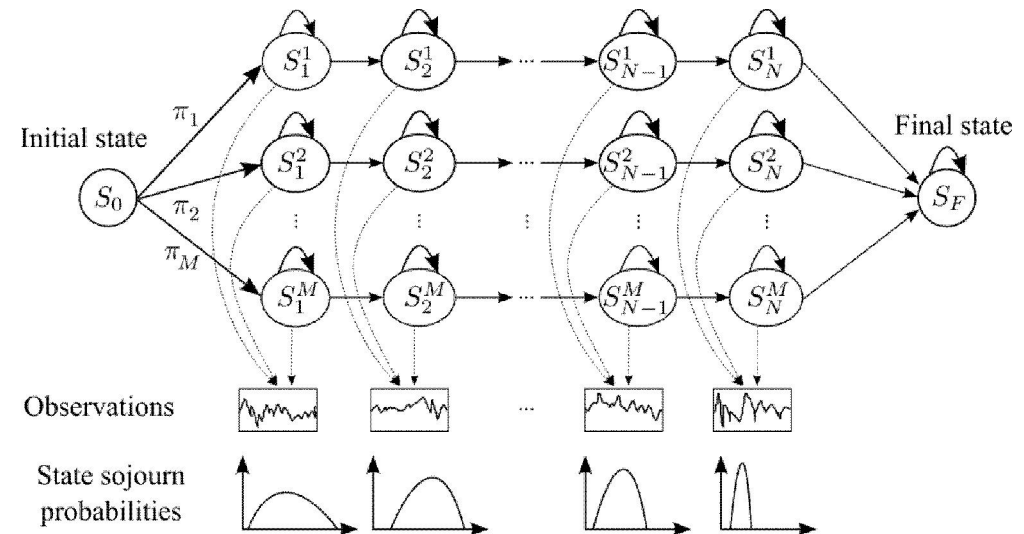
Assumptions

- Monotonic deterioration: left-right topology
- Initial and final states: non-emitting states
- Exclusive deterioration modes once initiated => No branches switching

Multi-branch discrete-state modeling

Multi-branch Hidden Markov Model

- Each branch \sim left-right HMM
 - Markovian property
- \Rightarrow state sojourn time \sim exponential
or geometrical distributions
- \Rightarrow May not be true in practice

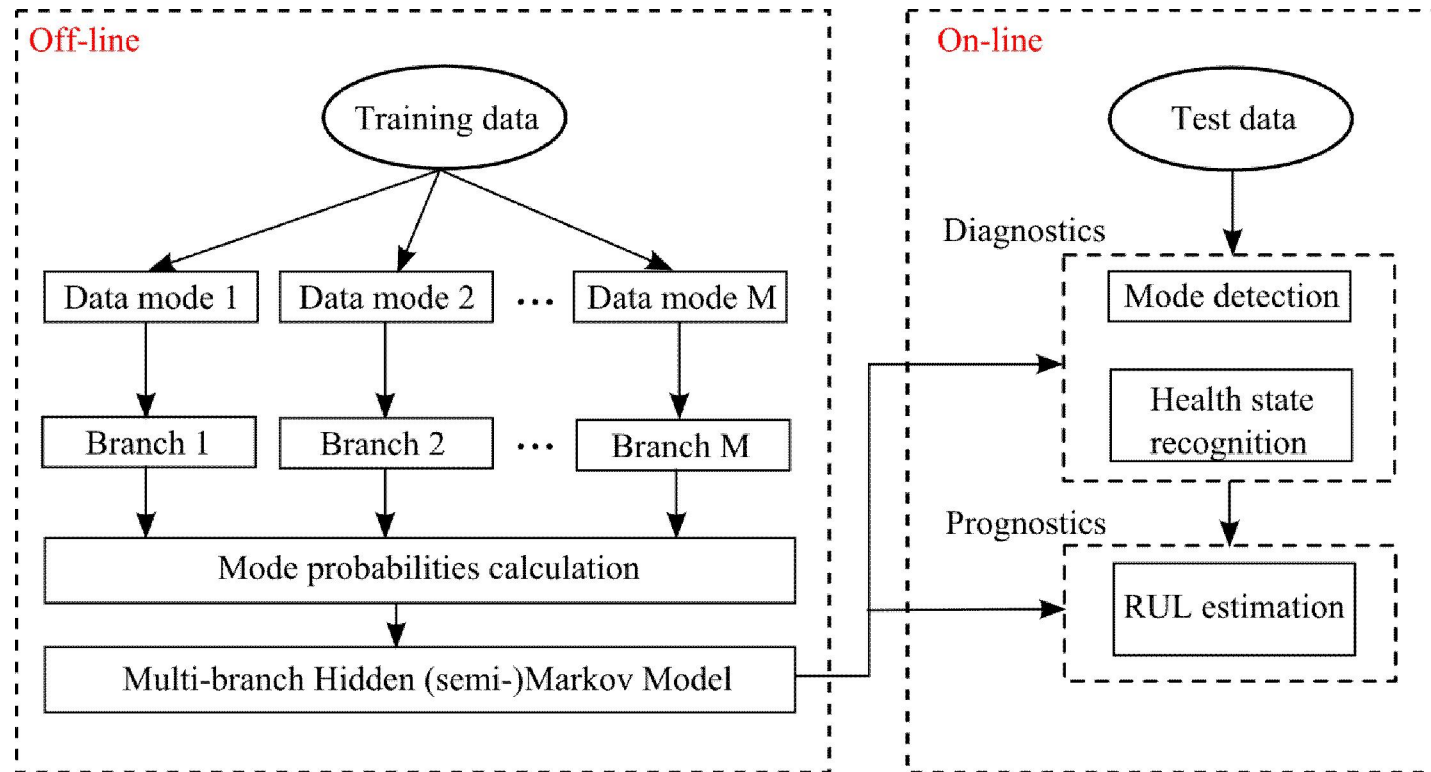


Multi-branch Hidden semi-Markov Model

- Each branch \sim left-right HsMM
 - Semi-Markov property: relax the Markovian assumption
- \Rightarrow allow arbitrary distributions for state sojourn time: Gaussian, Weibull, ...

Diagnostics and prognostics framework

Two-phase implementation: offline & online



Off-line phase

Model training

- Training data: high-level features
- Data classification => train each branch separately
 - MB-HMM: Baum-Welch algorithm adaption
 - MB-H^sMM: Forward-Backward procedure adaption [Yu06]
- *A priori* mode probabilities

$$\pi_k = P(\lambda_k) = K_k / K, \quad k = 1 \dots M$$

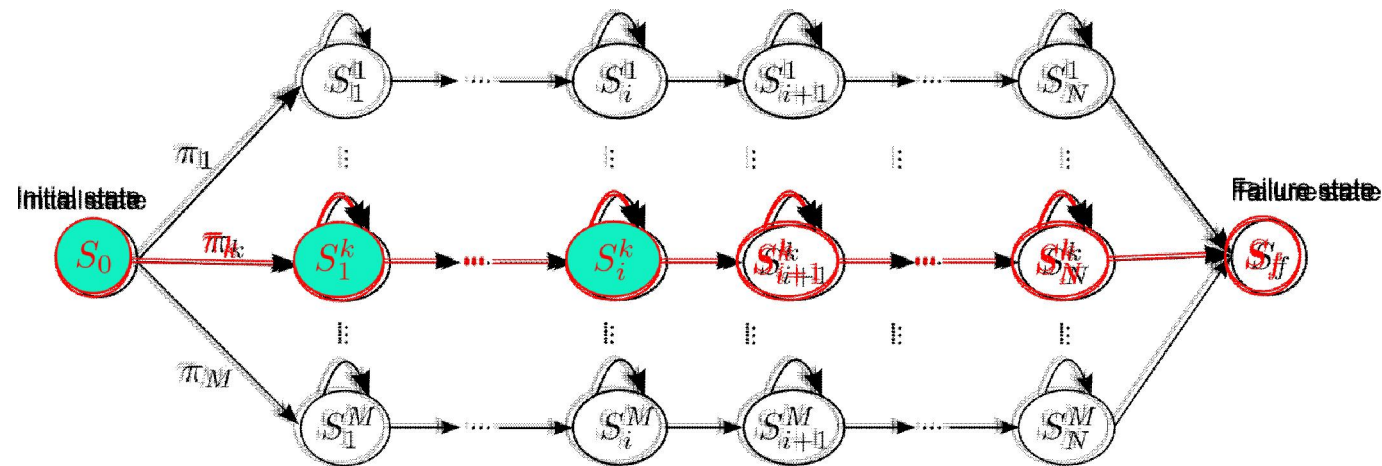
K_k : number of training sequences corresponding to the mode k

*[Yu06]: Practical implementation of an efficient forward-backward algorithm for an explicit-duration hidden Markov model. *IEEE Transactions on Signal Processing*, 54(5), 1947-1951.

On-line phase

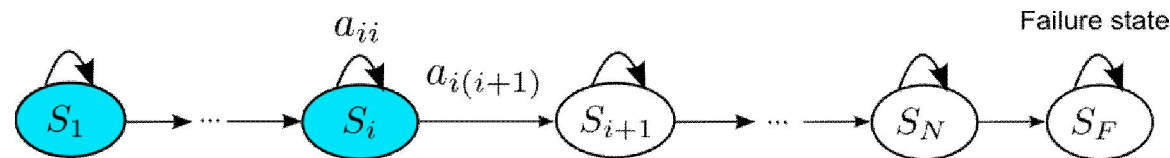
Diagnostics

- Mode detection: $\hat{k} = \arg \max_k P(\lambda_k | \mathbf{O})$
- Health-state assessment: Viterbi algorithm
 - Determine the “best” state sequence: $Q^* = \arg \max_{Q_k} P(\mathbf{O}, Q_k | \lambda_{\hat{k}})$
 - Actual health state = last state in Q^*



RUL estimation

One branch (HMM case)



- Suppose that the system is following the mode k
- Discrete time: RUL = number of transition steps to reach S_f **for the 1st time**

$$RUL_i^{(l)} = P(RUL = l \mid q_t = S_i) = P(q_{t+l} = S_N, q_{t+l-1} \neq S_N, \dots, q_{t+1} \neq S_N \mid q_t = S_i)$$

- Strictly left-right: Given S_i , the system can either stay in S_i or jump to S_{i+1}

$$RUL_i^{(l)} = a_{ii}RUL_i^{(l-1)} + a_{i(i+1)}RUL_{i+1}^{(l-1)}$$

⇒ **Recursive computation**

RUL estimation (*cont.*)

One branch (HsMM case)

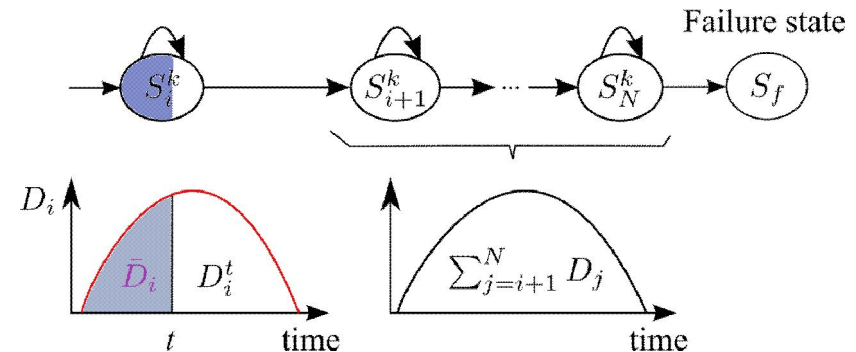
- Strictly left-right:

$$\text{RUL}_i^t = D_i^t + \sum_{j=i+1}^N D_j$$

D_j : sojourn time in states j

$D_i^t = D_i - \bar{D}_i \mid D_i > \bar{D}_i \sim$ truncated distribution

- Gaussian assumption: $\sum_{j=i+1}^N D_j \sim$ Normal distribution



Bayesian Model Averaging

- Take into account model uncertainty $P(\text{RUL} \mid \mathbf{O}) = \sum_{k=1}^M P(\text{RUL} \mid \lambda_k, \mathbf{O}) P(\lambda_k \mid \mathbf{O})$

where
$$P(\lambda_k \mid \mathbf{O}) = \frac{P(\mathbf{O} \mid \lambda_k) P(\lambda_k)}{\sum_{k=1}^M P(\mathbf{O} \mid \lambda_k) P(\lambda_k)}$$

RUL estimation (*cont.*)

Bayesian Model Averaging

- Take into account model uncertainty

$$P(\text{RUL} | \mathbf{O}) = \sum_{k=1}^M P(\text{RUL} | \lambda_k, \mathbf{O}) P(\lambda_k | \mathbf{O})$$

where $P(\lambda_k | \mathbf{O}) = \frac{P(\mathbf{O} | \lambda_k) P(\lambda_k)}{\sum_{k=1}^M P(\mathbf{O} | \lambda_k) P(\lambda_k)}$: Posteriori mode probability

Numerical examples

Data generation

- Fatigue Crack Growth (FCG) model: temporal evolution of a crack

- Stochastic version: $x_{t_i} = x_{t_{i-1}} + e^{w_{t_i}} C \left(\beta \sqrt{x_{t_{i-1}}} \right)^n \Delta t$

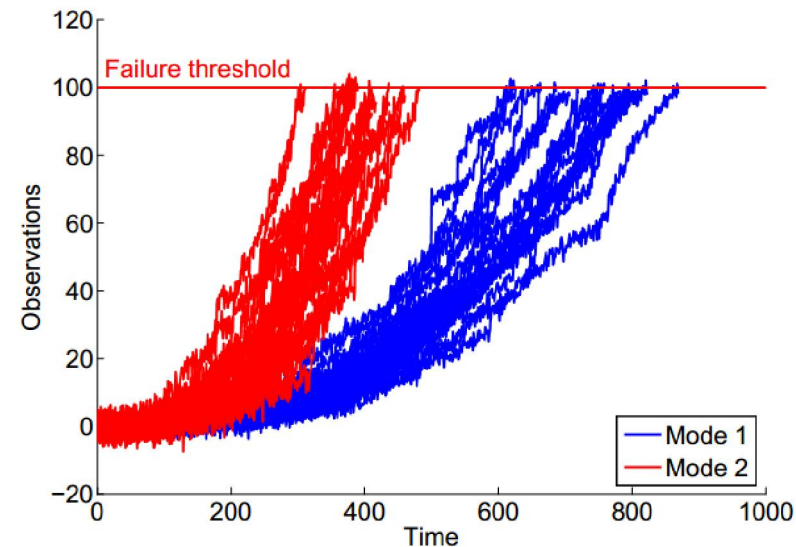
- Observation model: $y_{t_i} = x_{t_i} + \xi_{t_i}$

- Multi modes: $\beta(\varepsilon) = \beta_b \cdot e^{\gamma \varepsilon}$

γ_ε : environment factor

- Mode 1: $\gamma_1 = 0$ (slow)

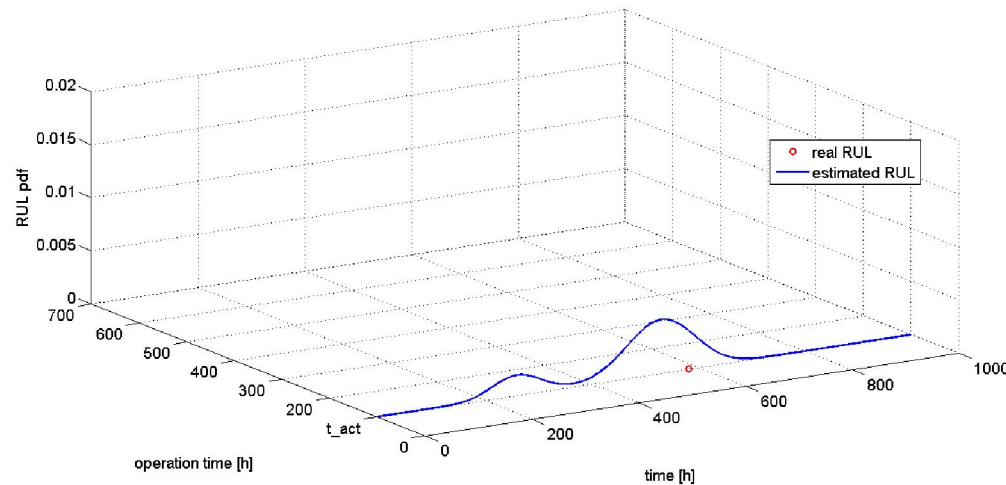
- Mode 2: $\gamma_2 = 0.75$ (quick)



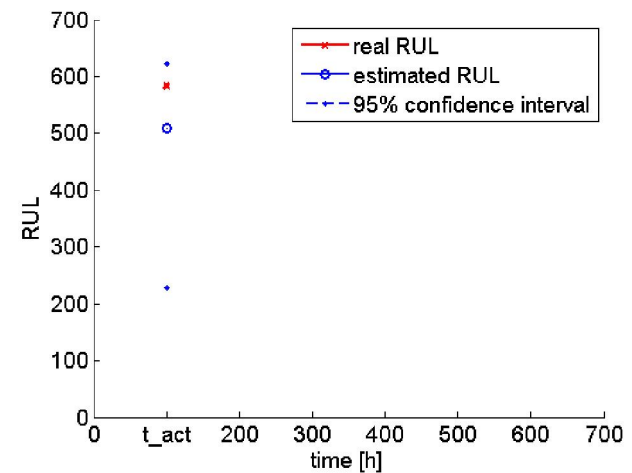
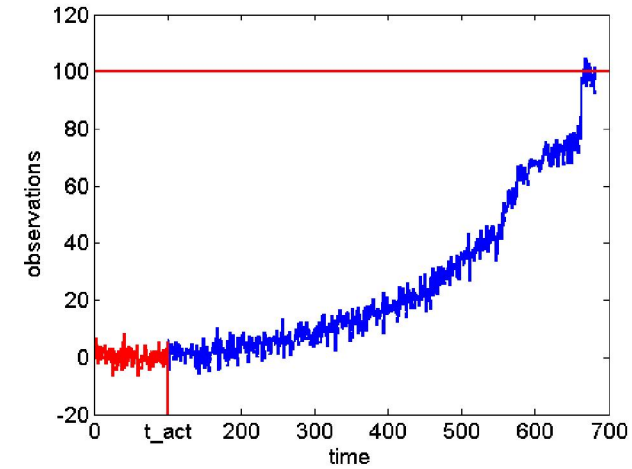
Two-mode deterioration data

Numerical examples

Online RUL estimation (MB-HsMM model)



RUL pdf

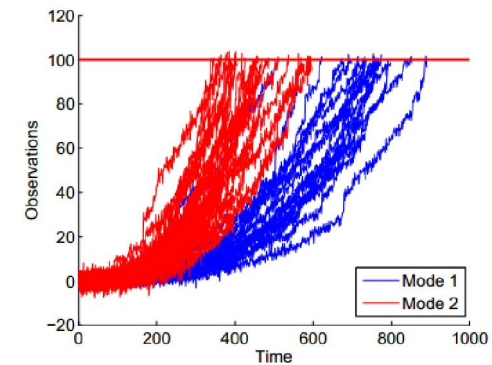
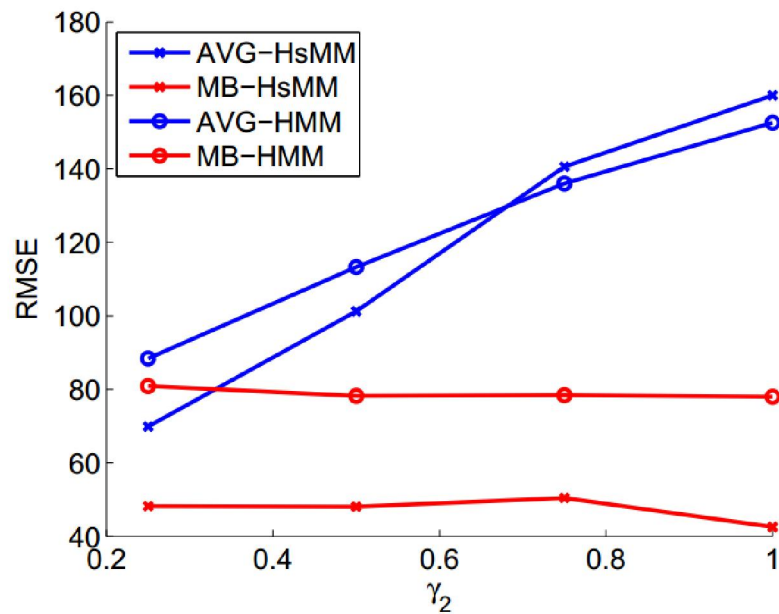


Mean value

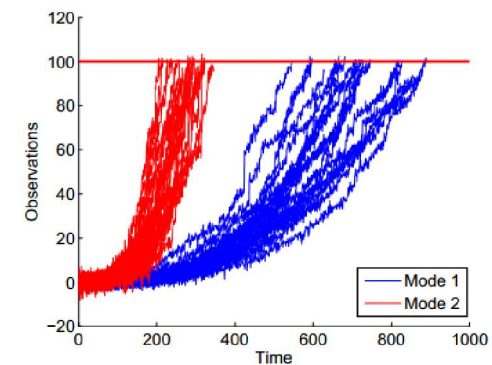
Numerical examples

Multi-branch vs. Average model

- Mode “distance”: difference between deterioration rates
- FCG case: mode distance $\sim \gamma_2$ (fix $\gamma_1 = 0$)
- Result



Small distance $\gamma_2 = 0.4$



Large distance $\gamma_2 = 0.8$

Case study (MB-HsMM model)

PHM08 competition

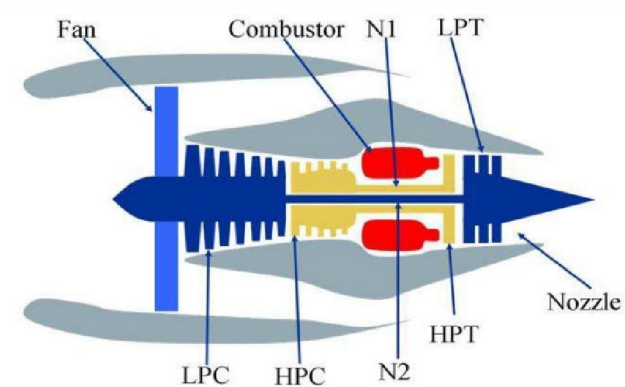
- C-MAPSS: large realistic commercial turbofan engine
- 2 data set: training & test
- One set: 218 identical and independent units
- Objective:
 - Construct a prognostic method basing on training data set
 - Use it to estimate the RUL of each unit in test data set

- Evaluation criterion:

$$S = \sum_{i=1}^{218} S_i$$

$$S_i = \begin{cases} e^{-d_i/13} - 1, & d_i \leq 0 \\ e^{d_i/10} - 1, & d_i > 0 \end{cases} : \text{penalty score}$$

$$d_i = RUL_{est}^i - RUL_{real}^i$$



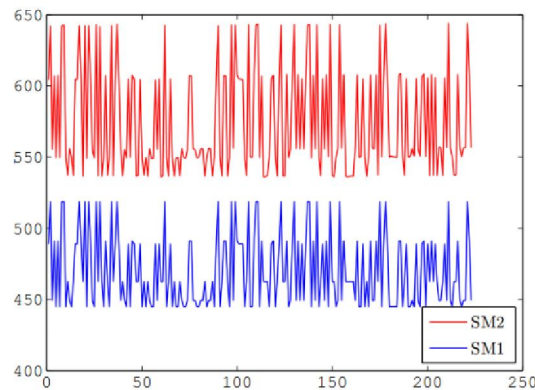
Simplified diagram of engine simulated in C-MAPSS

*C-MAPSS: Commercial Modular Aero-Propulsion System Simulation

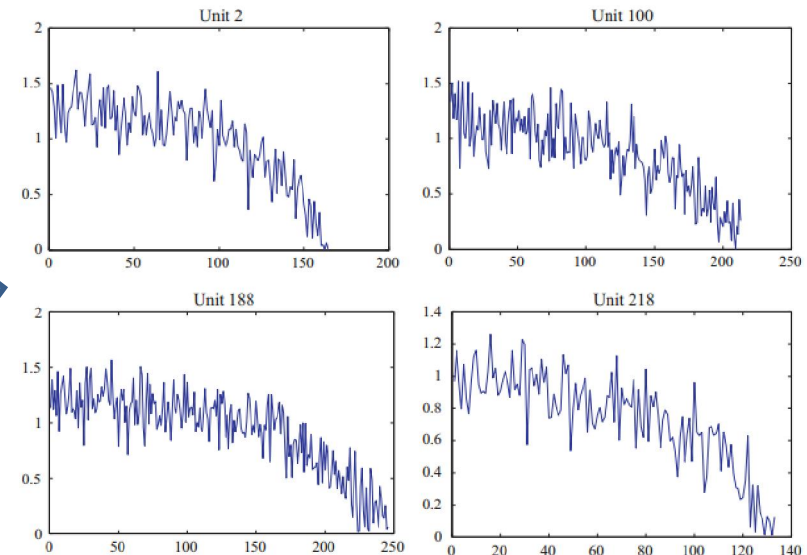
PHM08 data

Health indicator construction

- From [Le Son *et al.*]*



PCA-based



Health indicator evolution

- Clear tendency
- Better score than competition winners

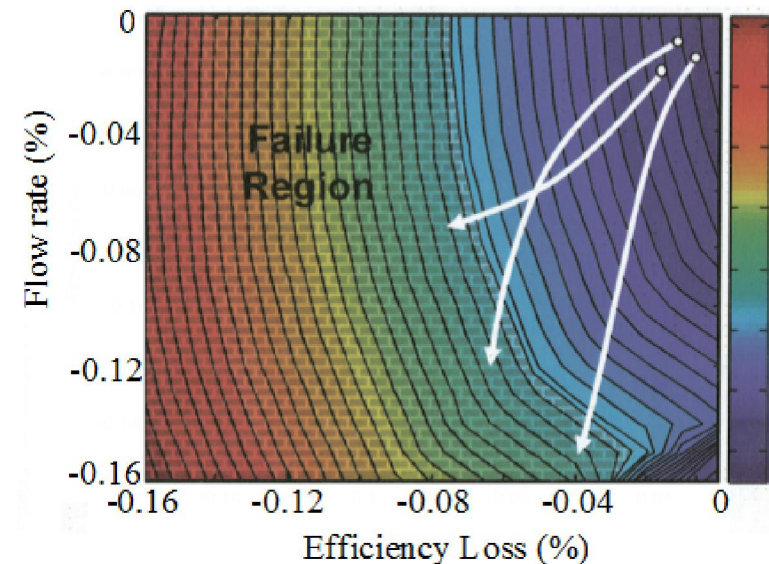
* [Le Son *et al.*] Remaining useful life estimation based on stochastic deterioration models: A comparative study. *Reliability Engineering & System Safety* 2012

Application of the MB-HsMM model

Number of deterioration modes

- Different fault propagation trajectories depending on the decrease rates of the flow rate (f) and efficiency (e) parameters
- 3 scenarios

	2 modes	3 modes	4 modes
Mode 1	$f < e$	$f < e$	$f \ll e$
Mode 2	$f > e$	$f \approx e$	$f < e$
Mode 3		$f > e$	$f > e$
Mode 4			$f \gg e$
No branches	2	3	4



Fault propagation trajectories

Application of the MB-HsMM model

Observations: mixture Gaussian model

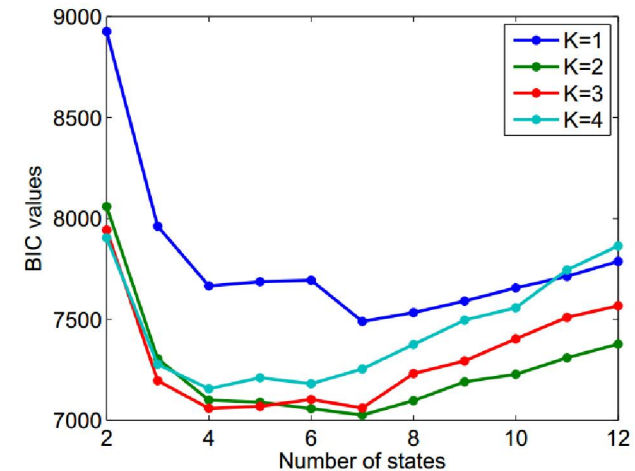
$$b_j(\mathbf{x}) = \sum_{k=1}^K c_{jk} \mathcal{N}(\mathbf{x}; \mu_{jk}, \Sigma_{jk})$$

Topology selection

➤ BIC criterion: $N = 7$; $K = 2$

RUL estimation result

Method	Score	RSE	MSE
1-branch HsMM	12246	502	1157
2-branch HsMM	6456	451	936
3-branch HsMM	5458	410	773
4-branch HsMM	3791	389	694
Wiener-based method	5575	423	823
Gamma-based method	4107	434	864



BIC values

Multi-branch discrete-state modeling

Summary

- Discrete health states => easy to interpret
- Multi-branch: take into account the co-existence of several deterioration modes
- Better RUL estimation performance
- **No mode switching** once initiated !

Outline

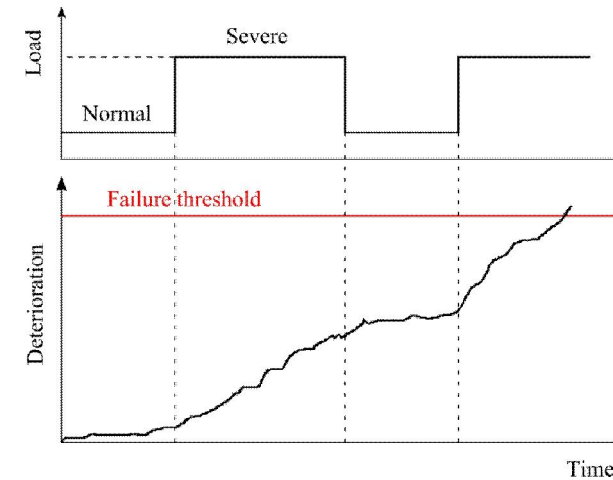
- Problem statement
- Multi-branch discrete-state models
- **Jump Markov linear systems**
- Conclusion & Perspectives

Switching state-space model

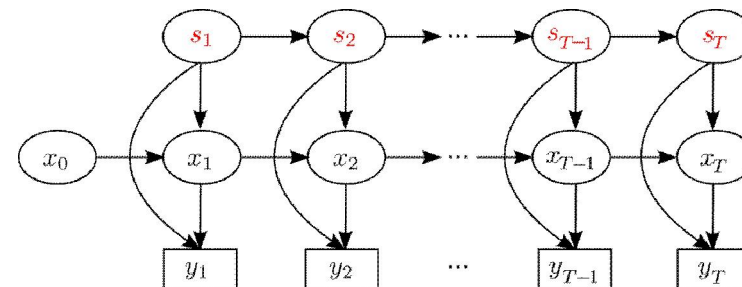
- State-space representation
 - Load-dependent deterioration
- ⇒ Deterioration modes co-exist in competition
- Mode switching ~ Markov jumps

$$\begin{cases} x_t = f(x_{t-1}, \omega_t, s_t) \\ y_t = g(x_t, v_t, s_t) \end{cases}$$

s_t : realization at time t of discrete variable S



Load-dependent deterioration



Switching state-space model

Jump Markov Linear System

- Assumption: deterioration dynamic can be **approximated by linear model**

$$\begin{cases} x_t = A_{s_t} x_{t-1} + \omega_t \\ y_t = C_{s_t} x_t + v_t \end{cases} \quad \begin{cases} \omega_t \sim \mathcal{N}(0, Q_{s_t}) \\ v_t \sim \mathcal{N}(0, R_{s_t}) \end{cases}$$

- M deterioration modes $s_t \in \{1, 2, \dots, M\}$

- Mode transition \sim discrete-time Markov chain

- Transition matrix:

$$\Pi = \begin{pmatrix} \pi_{11} & \pi_{12} & \dots & \pi_{1M} \\ \pi_{21} & \pi_{22} & \dots & \pi_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{M1} & \pi_{M2} & \dots & \pi_{MM} \end{pmatrix}$$

- Initial state: $\pi_1(i) = P(s_1 = i)$

- Identifiability guarantee: C are fixed

Parameters learning problem

Model parameters

$$\Theta = \left\{ (A_i, Q_i, R_i)_{i=1, \dots, M}, \mu_0, \Sigma_0, \Pi, \pi_1 \right\}$$

➤ X, S are hidden => Expectation-Maximization algorithm

○ E step: $Q(\Theta | \Theta^{(k)}) = \mathbf{E} \left[\log \mathbb{P}(\mathcal{X}_T, \mathcal{S}_T, \mathcal{Y}_T | \Theta) | \mathcal{Y}_T, \Theta^{(k)} \right]$

○ M step: $\Theta^{(k+1)} = \arg \max_{\Theta} Q(\Theta | \Theta^{(k)})$

➤ Problem: Presence of switching dynamic

$$Q(\Theta | \Theta^{(k)}) = \sum_{\mathcal{S}_T} \left(\mathbb{P}(\mathcal{S}_T | \mathcal{Y}_T, \Theta^{(k)}) \int p(\mathcal{X}_T | \mathcal{S}_T, \mathcal{Y}_T, \Theta^{(k)}) \log \mathbb{P}(\mathcal{X}_T, \mathcal{S}_T, \mathcal{Y}_T | \Theta) d\mathcal{X}_T \right)$$

➤ Computed over all possible sequences of discrete states \mathcal{S}_T => Intractable

=> **Approximated EM algorithm**

Approximated EM algorithm

Pruning technique

- Idea: Calculate Q over the most “likely” state sequence
=> Adaption of the Viterbi algorithm
- Do not guarantee the convergence, but still sufficient in several practical cases

Approximated Q function

$$Q(\Theta | \Theta^{(k)}) \approx \int p(\mathcal{X}_T | \mathcal{S}_T^*, \mathcal{Y}_T, \Theta^{(k)}) \log \mathbb{P}(\mathcal{X}_T, \mathcal{S}_T^*, \mathcal{Y}_T | \Theta) d\mathcal{X}_T$$

\mathcal{S}_T^* : the most likely state sequence

=> Calculated by Rauch-Tung-Streiber (RTS) smoother

JMLS based diagnostics

Mode probabilities

- Given test data until time t

$$\mu_t(i) = \mathbb{P}(s_t = i) = \frac{\mathbb{P}(\mathcal{S}_{t,i}^*)}{\sum_{i=1}^M \mathbb{P}(\mathcal{S}_{t,i}^*)} = \frac{1}{1 + \exp\left(\sum_{j \neq i} J_{t,j} - J_{t,i}\right)}$$

$\mathcal{S}_{t,i}^*$ the best state sequence at time t that ends in i

Health state assessment

- Mixture of mode-dependent states

$$\begin{cases} \hat{x}_t = \sum_{i=1}^M \mu_t(i) \hat{x}_{t,i} \\ \hat{\Sigma}_t = \sum_{i=1}^M \mu_i(t) \Sigma_{t,i} + \sum_{i=1}^M \mu_t(i) (\hat{x}_{t,i} - \hat{x}_t)(\hat{x}_{t,i} - \hat{x}_t)^T \end{cases}$$

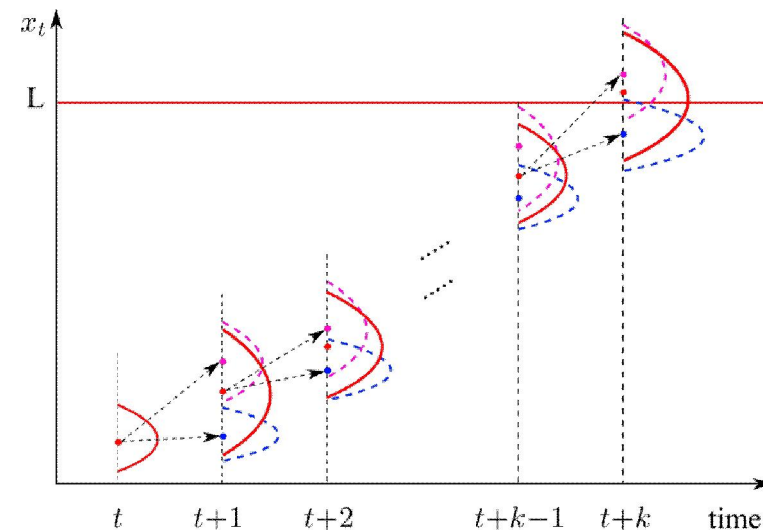
RUL prediction

- Discrete time: $RUL = (\min k \geq 1 : x_{t+k} \geq L \mid x_t < L)$
- Mode switching: M fold increase in number of Gaussian distributions
- ⇒ Intractable computation
- ⇒ Approximation: merge all one-step predicted Gaussian distributions into one

$$p(x_{t+1}|t) \approx \sum_{i=1}^M \mu_{t+1}(i) \mathcal{N}(x_{t+1}|t,i, \Sigma_{t+1}|t,i)$$

- Mode probability update

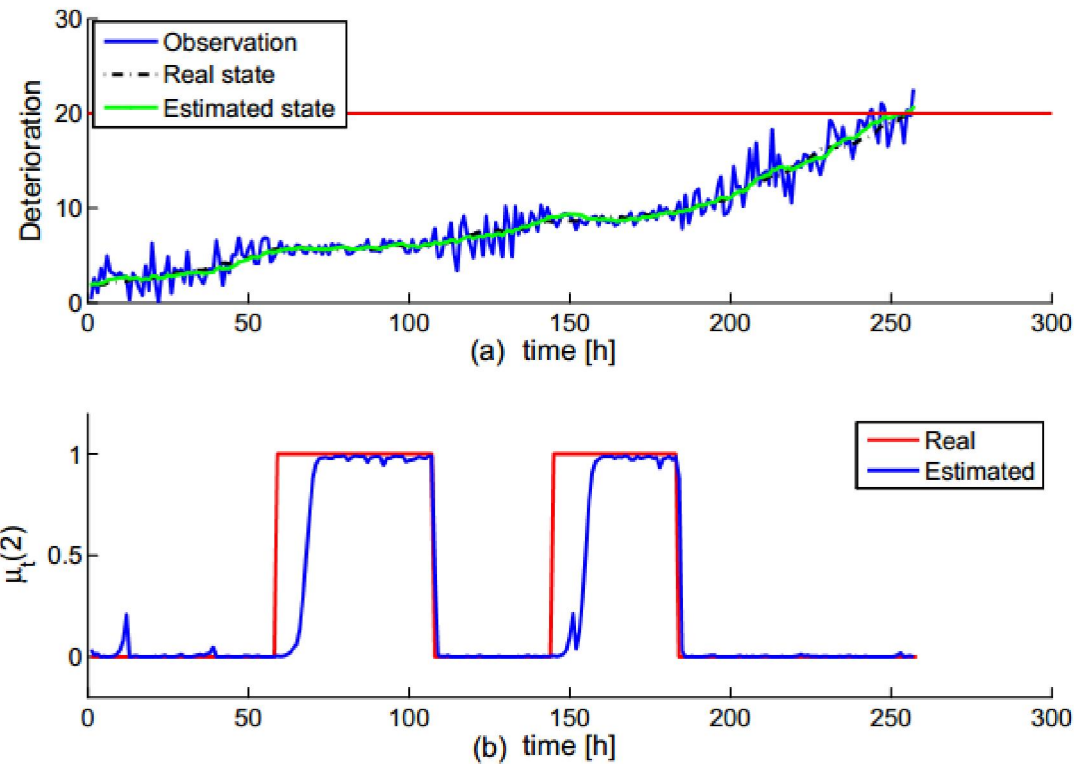
$$\mu_{t+1}(i) = \sum_{j=1}^M \pi_{j,i} \cdot \mu_t(j)$$



RUL prediction illustration for 2-mode case

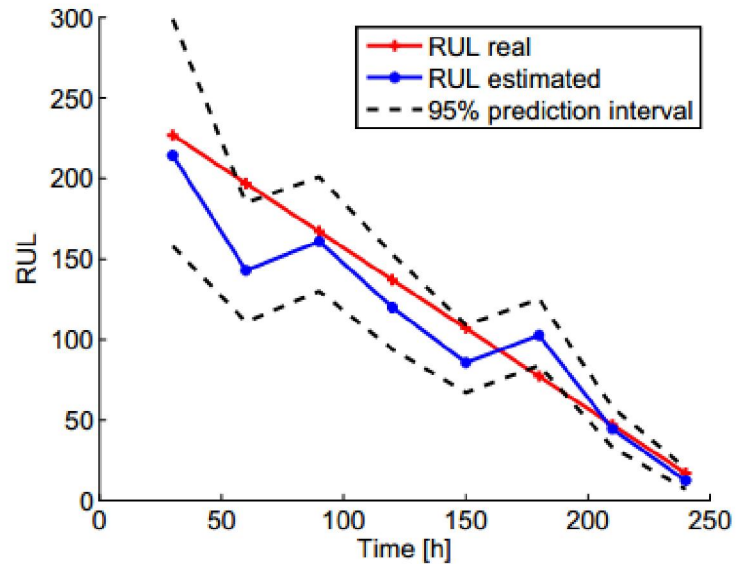
Diagnosis result

Mode detection and health assessment

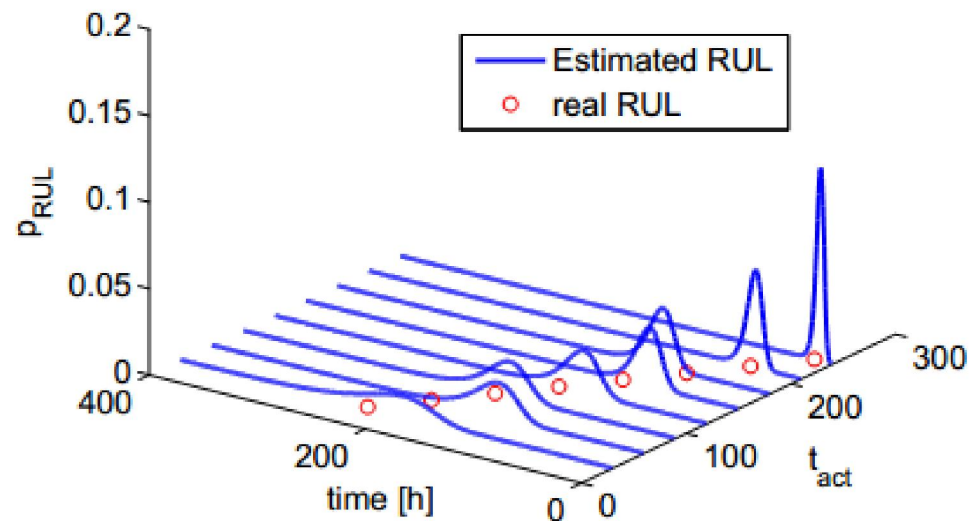


RUL estimation result

Online prediction



Mean estimated RUL at different times



Evolution of estimated RUL pdf

Outline

- Problem statement
- Multi-branch discrete-state models
- Jump Markov linear systems
- Conclusion & Perspectives

Conclusion & Perspectives

Conclusion

- Co-existence problem of multiple deterioration modes: multi-branch modeling
 - Discrete states: MB-HMM & MB-HsMM
 - Continuous states: JMLS
- Development of associated diagnostics & prognostics framework
 - Actual deterioration mode detection
 - Current health status assessment
 - RUL estimation

Conclusion & Perspectives

Perspectives

- MB-HMM & MB-HsMM models
 - Relax the left-right assumption
 - Allow mode switching
- JMLS model
 - Extension to non-linear models
 - Improvement of model learning methods
 - Semi-Markov mode transitions
- Dynamic adaptation of maintenance strategies based on RUL estimation results
- Models application with real-life data



Thank you for your attention!

