Degradation-Based Maintenance Using Stochastic Filtering for Systems under Imperfect Maintenance

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Intro: main line

Maintenance actions can be classified, according to the efficiency, into 3 types:

- perfect maintenance, with each maintenance leaving the system as if it were new;
- minimal maintenance, with each maintenance leaving the system in the condition as it was just before the maintenance;
- imperfect maintenance, with each maintenance restoring a system's condition to a younger state but not as good as new.

Compared with the "perfect" and the "minimal" assumptions, it is more realistic that maintenance actions are imperfect.

Intro: main line

How to mathematically quantify the effect of each maintenance?

- perfect maintenance: renewal process;
- minimal maintenance: non-homogeneous poisson process;
- imperfect maintenance: various treatments (Wang 2002).

Intro: main line

One of the most popular treatments is to invoke the hazard rate function:

improvement-factor method

Let $h(t),\ t \geq 0$, denote the hazard rate function (**monotonically increasing**) of the target system. Right after a maintenance action at time $t_1 \geq 0$, the hazard rate function changes into $bh(t - t_1 + at_1)$.

- $0 < a < 1$ is an age-reduction factor.
- $b > 1$ is a hazard-rate-increase factor.

Intro: main line

interpretation:

If an imperfect maintenance action is taken at time t_1, the hazard rate right after the maintenance action changes to $bh(at_1)$.

- There is a decrease in the age of the system ($at_1 < t_1$) and thus a decrease in the hazard rate, indicating that the system becomes younger.
- After the maintenance, the hazard rate increases faster ($b > 1$).

As time elapses, the hazard rate function has the form $bh[(t - t_1) + at_1]$.

- $(t - t_1)$ is the time elapsed from the last maintenance.

Likewise, after the second maintenance at time t_2, the hazard rate function has the form

$$b^2 h\{(t - t_2) + a[(t_2 - t_1) + at_1]\}.$$

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A non-stationary Wiener process, $\{X_t,\ t \geq 0\}$, with drift function $v(t)$ and variance parameter σ^2 can be expressed as $X_t = v(t) + \sigma B_t$.

- $v(t)$ is a monotonically increasing, right-continuous, real-valued function on $t \geq 0$ with $v(0) = 0$.
- $\{B_t, t \geq 0\}$ is the standard Brownian motion.

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How to mathematically characterize the effect of each imperfect maintenance **in the context of degradation-based maintenance**?

The improvement-factor method is ineffective since the hazard rate function is extremely complex and non-analytical.

Intro: main line

- For the non-stationary Wiener process $X_t = v(t) + \sigma B_t$, the expected degradation up to time t is $E(X_t) = v(t)$.
- The first-order derivative of $v(t)$, $v'(t)$, characterizes the deteriorating speed/rate, termed as **degradation rate function**.

  lubrication: The lubricating activity has its impact on the degradation rate function, slowing down the wearing process.
- The concept of the improvement factor method can be extended to the degradation rate function $v'(t)$ to model maintenance efficiency.

Intro: main line

We assume that if an imperfect maintenance action is taken at time $t_1 \geq 0$, the degradation rate function $v'(t)$ after the maintenance has the form $bv'[(t - t_1) + at_1]$. Here $0 < a < 1$ is an age-reduction factor, and $b > 1$ is a degradation-rate-increase factor.

- If $a = b = 1$, we arrive at the minimal assumption.
- If $a = 0$ and $b = 1$, we arrive at the perfect assumption.

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Two main advantages:

- Deriving the hazard rate function via the first hitting time distribution function is mathematically intractable, especially when the drift function $v(t)$ is non-linear.
- The impact of maintenance actions on the hazard rate is unmeasurable. The impact on the degradation rate can be exteriorized from consecutive degradation measurements.

An illustrative example: theory

Having built the model to characterize maintenance efficiency, the problem reduces to: **How to evaluate the impact factors a and b?**

Illustrative example:

$X_t = \lambda t^{\theta} + \sigma B_t$, with degradation rate function $v'(t) = \lambda\theta t^{(\theta-1)}$.

An illustrative example: theory

- To simplify matters, we assume that the degradation rate function immediately after a maintenance at time t changes from $v'(t)$ to $bv'(t)$ (no a).
- By re-writing

$$bv'(t) = b\lambda\theta t^{(\theta-1)}$$

as

$$bv'(t) = \tilde{\lambda}\theta t^{(\theta-1)},$$

where $\tilde{\lambda} = b\lambda$, we can state that the maintenance activity has its effect on the scale parameter: Right after the maintenance the degradation rate function changes from

$$v(t) = \lambda\theta t^{(\theta-1)}$$

to

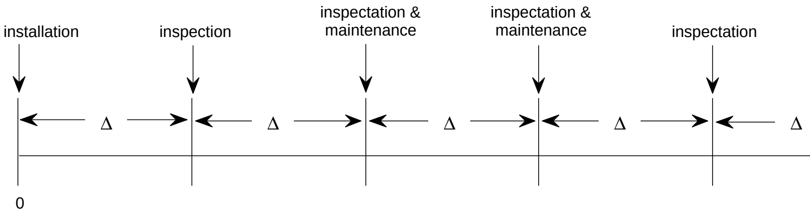
$$v(t) = \tilde{\lambda}\theta t^{(\theta-1)}.$$

An illustrative example: theory

To quantify the effectiveness of each maintenance action, we need to estimate the value of the improvement factor b after each maintenance.

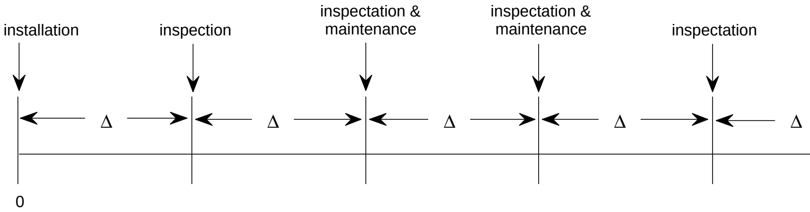
To estimate the value of the improvement factor b after each maintenance, it is equivalent **just to assess the value of the scale parameter λ after each maintenance**.

An illustrative example: theory



Degradation-measuring points are equally spaced with $\triangle > 0$ being the constant time between two consecutive degradation-measuring points, i.e., measuring at epochs $i\triangle$ $(i = 1, 2, 3, \ldots)$.

An illustrative example: theory



At each degradation-measuring point we

- first, measure the degradation of the maintaining system;
- second, assess the value of the scale parameter based on the history of maintenance actions and degradation measurements;
- third, decide whether or not to take maintenance action (I will not talk).

An illustrative example: theory

- Denote x_i to be the degradation measurement at time $i\triangle$ with $x_0 = 0$.
- Denote λ_i to be the hidden true value of the scale parameter at time $i\triangle$ before any maintenance action.

By assuming that the improvement factor b is a random variable following the normal distribution $N\left(\bar{b}, Q\right)$, we have process equation

$$\lambda_i = b_{i-1}\lambda_{i-1} + w_{i-1}, \tag{3.1}$$

and measurement equation

$$x_i - x_{i-1} = \eta_i\lambda_i + \omega_i, \tag{3.2}$$

in which b_i and η_i are known coefficients; the process noise w_i and measurement noise ω_i are both Gaussian white.

An illustrative example: theory

Eq. (3.1) and Eq. (3.2) construct a Kalman filter. At the time $i\,\triangle$, given the new measurement x_i, we define

- $\hat{\lambda}_i$ to be the a posterior estimate of the true value λ_i,
- P_i to be the a posterior error variance of the estimate $\hat{\lambda}_i$.

Hence, to recursively estimate the scale parameter, the Kalman filter the following updating equations:

$$\hat{\lambda}_i = b_{i-1}\hat{\lambda}_{i-1} + \frac{(b_{i-1}^2 P_{i-1} + Q_{i-1})\eta_i}{(b_{i-1}^2 P_{i-1} + Q_{i-1})\eta_i^2 + \sigma^2\,\triangle} \times \left(y_i - \eta_i b_{i-1}\hat{\lambda}_{i-1}\right), ($$

$$P_i = \frac{(b_{i-1}^2 P_{i-1} + Q_{i-1}) \times \sigma^2\,\triangle}{(b_{i-1}^2 P_{i-1} + Q_{i-1})\eta_i^2 + \sigma^2\,\triangle}. \tag{3.4}$$

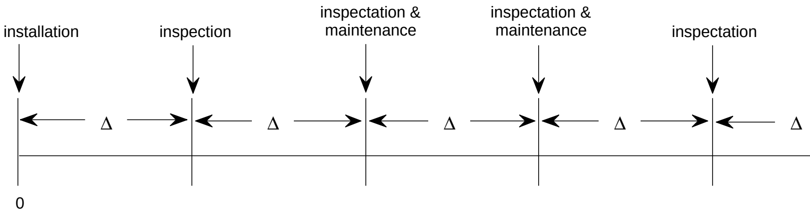
An illustrative example: theory

Summation:

- To evaluate the effect of the $(i-1)$th maintenance, we only need to assess, at time $i\,\triangle$, the value of the scale parameter λ_i.
- $\hat{\lambda}_i$ is the estimate on the scale parameter λ_i, and P_i characterizes the accuracy of the estimate $\hat{\lambda}_i$.
- $\hat{\lambda}_i$ and P_i can be easily obtained by using equations (3.3) and (3.4).

Problem is solved!

An illustrative example: simulated data $\{x_1, x_2, ..., x_i, ...\}$



Degradation-measuring points are equally spaced with $\triangle > 0$ being the constant time between two consecutive degradation-measuring points, i.e., measuring at epochs $i\,\triangle\ \ (i = 1, 2, 3, ...)$.

An illustrative example: simulated data $\{x_1, x_2, ..., x_i, ...\}$

At the first point $\triangle$, simulate $x_1 \sim N(\lambda \triangle^{\theta}, \ \sigma^2 \triangle)$.

- If a maintenance activity is then taken at epoch $\triangle$, the value of the scale parameter immediately after the maintenance changes to $b_1\lambda$, with $b_1 \sim N\left(\bar{b}, Q\right)$. Therefore, at the second point $2\triangle$, simulate a degradation increment

$$y_2 \sim N\left(b_1\lambda(2^{\theta} - 1) \triangle^{\theta}, \ \sigma^2 \triangle\right).$$

- If no maintenance is taken at epoch $\triangle$, at the second point simulate a degradation increment

$$y_2 \sim N\left(\lambda(2^{\theta} - 1) \triangle^{\theta}, \ \sigma^2 \triangle\right).$$

The second observation of degradation is hence $x_2 = x_1 + y_2$.

An illustrative example: simulated data $\{x_1, x_2, ..., x_i, ...\}$

By analogy, at the ith point, $i = 2, 3, 4, ...$, simulate a degradation increment

$$y_i \sim N\left(\prod_{j=1}^{i-1} b_j^{m_j} \lambda \left[i^\theta - (i-1)^\theta\right] \triangle^\theta,\ \sigma^2 \triangle\right),$$

and the ith degradation measurement is set to be $x_i = x_{i-1} + y_i$. Here m_i $(i = 1, 2, 3, ...)$ are the indicators with $m_i = 1$ indicating that the system is maintained at the ith point.

An illustrative example: results

Apply the proposed stochastic filter on the simulated data $\{x_1, x_2, ..., x_i, ...\}$.

Since the underlying true value of the scale parameter at each point is known, we calculate the value of the a posterior error variance P_i and the value of the deviation $\hat{\lambda}_i - \lambda_i$.

The evolutions of $\{\hat{\lambda}_i - \lambda_i, \ i = 1, 2, 3...\}$ and $\{P_i, \ i = 1, 2, 3...\}$ are depicted in the following figure.

An illustrative example: results

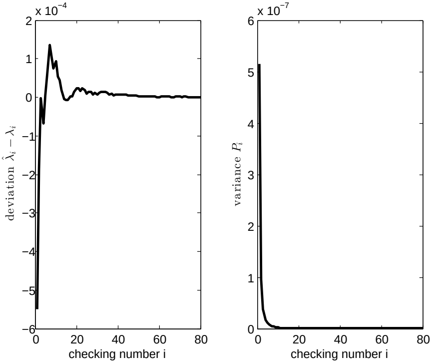


Figure: The evolutions of $\hat{\lambda}_i - \lambda_i$ and P_i.

The deviation and the error variance converge to zero rapidly, showing the efficiency of the proposed algorithm.

An illustrative example: results

To show the robustness of the algorithm, we run the simulation for 100 times. The 100 evolving paths of the deviation $\hat{\lambda}_i - \lambda_i$ and 100 evolving paths of the error variance P_i are plotted in the following figure.

An illustrative example: results

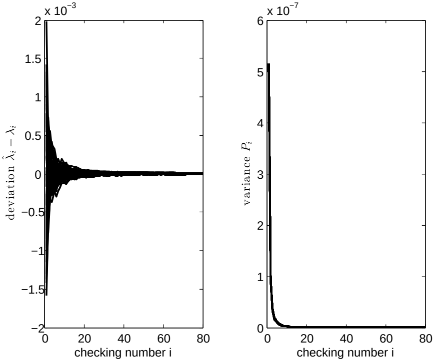


Figure: Evolution paths of $\{\hat{\lambda}_i - \lambda_i, \ i = 1, 2, 3...\}$ and $\{P_i, \ i = 1, 2, 3...\}$ with 100 samples.

All the deviations and variances converge rapidly to zero, showing the robustness of the stochastic filtering algorithm.

Open topics

Future research can be carried out in

- implementing this strategy into various condition-based maintenance schemes,
- studying the case in which both the age-reduction factor and the degradation-rate-increase factor are involved,
- dealing with other degradation processes via stochastic filtering technique.